Question 1

A curve has equation $y = x^3 - 3x^2 + 3x$.

Calculate the gradient of the curve at $x = 4$.

$f'(x) = 3x^2 - 6x + 3$

1 Find the $f'(x)$.

The gradient of the curve at $x = 4$ is $f'(4)$

2 Substitute the x in the $f'(x)$ with the x-coordinate of the point.

$f'(4) = 3(4)^2 - 6(4) + 3$

$= 27$

Question 2

A curve has equation $y = x(1 - x)(2 - x)$.

Calculate the gradient of the curve at $x = 0$.

$$y = x(1 - x)(2 - x)$$

$$y = (x - x^2)(2 - x)$$

$$y = 2x - x^2 - 2x^2 + x^3$$

$$y = x^3 - 3x^2 + 2x$$

$$f'(x) = 3x^2 - 6x + 2$$

The gradient of the curve at $x = 0$ is $f'(0)$

$$f'(0) = 3(0)^2 - 6(0) + 2$$

$$= 2$$

1 Find the $f'(x)$.

2 Substitute the x in the $f'(x)$ with the x-coordinate of the point.

Question 3

A curve has equation $y = 4x - \frac{1}{x}$.

Calculate the gradient of the curve at $x = -2$.

$$y = 4x - x^{-1}$$

1 Find the $f'(x)$.

$$f'(x) = 4 + x^{-2}$$

The gradient of the curve at $x = -2$ is $f'(-2)$

2 Substitute the x in the $f'(x)$ with the x-coordinate of the point.

$$f'(-2) = 4 + (-2)^{-2}$$

$$= 4 + \frac{1}{(-2)^2} = 4.25$$

Question 4

Find the gradient of the curve $y = 3x^2 - 5x + 2$ at the point where $x = 2$.

Differentiate: $\frac{dy}{dx} = 6x - 5.$

Substitute $x = 2$: $\frac{dy}{dx} = 6(2) - 5 = 12 - 5 = 7.$

Gradient = 7.

Question 5

Find the gradient at $x = 1$ for $y = 3x^3 - x^2 + 5x - 2$.

Differentiate: $\frac{dy}{dx} = 9x^2 - 2x + 5$.

Substitute $x = 1$: $\frac{dy}{dx} = 9(1)^2 - 2(1) + 5 = 9 - 2 + 5 = 12$.

Gradient = 12.

Question 6

A curve has equation $y = x^5 - kx^4 + 3$, where k is a nonzero constant.
The point p, whose x-coordinate is -1, lies on the curve.

Determine, the gradient of the curve at point P, in terms of k.

$f'(x) = 5x^4 - 4kx^3$

1. Find the $f'(x)$.

The gradient of the curve at $x = -1$ is $f'(-1)$.

2. Substitute the x in the $f'(x)$ with the x-coordinate of the point.

$f'(-1) = 5(-1)^4 - 4k(-1)^3$

$f'(-1) = 5 + 4k$

The gradient of the curve at point P, in terms of k is $5 + 4k$.

Question 7

A curve has equation $y = x^4 - 2x^3 + 3$.

Find the x-coordinate of the points on the graph, where the gradient is 0.

$f'(x) = 4x^3 - 6x^2$

1 Find the $f'(x)$.

The gradient of the curve at x is $f'(x)$ and equal to 0.

2 Substitute the x in the $f'(x)$ with the x-coordinate of the point.

$$f'(x) = 4x^3 - 6x^2 = 0$$

$$2x^2(2x - 3) = 0$$

$2x^2 = 0 \qquad x = 0$

$(2x - 3) = 0 \qquad x = \frac{3}{2}$

The x-coordinate of the points on the graph, where the gradient is 0 are $x = 0$ and $x = \frac{3}{2}$.

Question 8

The curve has equation $y = ax^3 - bx^2 + 2$, where a and b are constant.
Point $A(2,6)$ lies on curve.

Given that gradient at A is -8, determine the values of a and b.

We have 2 unknown variables a and b, so we need to make two equations and then solve simultaneous equations.

First equation:

$A(2,6) \longrightarrow 6 = a(2)^3 - b(2)^2 + 2$

$$\boxed{4 = 8a - 4b}$$

Point A lies on the curve, so the coordinates of A satisfy the equation $y = ax^3 - bx^2 + 2$.

Second equation:

gradient at $A(2,6)$ is $-8 \longrightarrow f'(2) = -8$

$f'(x) = 3ax^2 - 2bx$

$f'_{(2)} = 3a(2)^2 - 2b(2) = -8$

$$\boxed{12a - 4b = -8}$$

$$\begin{cases} 8a - 4b = 4 \longrightarrow b = 2a - 1 \\ 12a - 4b = -8 \end{cases}$$

$12a - 4(2a - 1) = -8$

$4a = -12$

$a = -3 \longrightarrow b = 2a+1$

$b = 2(-3) + 1 = -5$

Values of a and b are $a = -3$ and $b = -5$.

Question 9

Find the equation of the tangent to the curve $y = x^2 + 3x - 5$ at the point where $x = 2$.

Substitute $x = 2$ into the equation of the curve to find y:

$$y = (2)^2 + 3(2) - 5 = 4 + 6 - 5 = 5$$

The point is (2,5).

The gradient of the tangent at a point is given by the derivative of the curve at that point.

$$\frac{dy}{dx} = 2x + 3$$

Substitute $x = 2$ into $\frac{dy}{dx}$:

$$2(2) + 3 = 4 + 3 = 7$$

Determine the equation of the tangent.
$y - y_A = m(x - x_A)$

Here, $m = 7$, $(x_1, y_1) = (2, 5)$.

Substitute:

$$y - 5 = 7(x - 2)$$

Simplify:

$$y - 5 = 7x - 14$$

$$y = 7x - 9$$

The equation of the tangent is

$$y = 7x - 9$$

Question 10

A curve is defined by $y = 4x^3 - x^2 + 2x + 1$. Determine the equation of the tangent to the curve at the point where $x = -1$.

Substitute $x = -1$ into the equation of the curve to find y:

$$y = 4(-1)^3 - (-1)^2 + 2(-1) + 1$$
$$y = -4 - 1 - 2 + 1 = -6$$

The point is $(-1, -6)$.

The gradient of the tangent at a point is given by the derivative of the curve at that point.

$$\frac{dy}{dx} = 12x^2 - 2x + 2$$

Substitute $x = -1$ into $\frac{dy}{dx}$:

$$\frac{dy}{dx} = 12(-1)^2 - 2(-1) + 2$$
$$\frac{dy}{dx} = 12 + 2 + 2 = 16$$

So, the gradient of the tangent is 16.

Determine the equation of the tangent.
$y - y_A = m(x - x_A)$

Here, $m = 16$, $(x_1, y_1) = (-1, -6)$.

Substitute:

$$y - (-6) = 16(x - (-1))$$

Simplify:

$$y + 6 = 16x + 16$$

$$y = 16x + 10$$

The equation of the tangent is

$$y = 16x + 10$$

Question 11

The equation of a curve is given as $y = \sin x$. Find the equation of the tangent to the curve at the point $(\frac{\pi}{2}, 1)$.

The point is $(\frac{\pi}{2}, 1)$.

The gradient of the tangent at a point is given by the derivative of the curve at that point.

$$\frac{dy}{dx} = \cos x$$

Substitute $x = \frac{\pi}{2}$:

$\frac{dy}{dx} = \cos\frac{\pi}{2} = 0$

So, the gradient of the tangent is 0.

Determine the equation of the tangent.
$y - y_A = m(x - x_A)$

Here, $m = 0$, $(x_1, y_1) = (\frac{\pi}{2}, 1)$.

Substitute:

$$y - 1 = 0(x - \frac{\pi}{2})$$

Simplify:

$$y - 1 = 0$$

$$y = 1$$

The equation of the tangent is

$$y = 1$$

Question 12

The curve $y = \frac{1}{x}$ passes through the point $(1,1)$. Calculate the equation of the tangent to the curve at this point.

The point is $(1, 1)$.

The gradient of the tangent at a point is given by the derivative of the curve at that point.

Rewrite as $y = x^{-1}$.

Differentiate:

$$\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$$

$$\frac{dy}{dx} = -\frac{1}{(1)^2} = -1$$

So, the gradient of the tangent is -1.

Determine the equation of the tangent.
$y - y_A = m(x - x_A)$

$m = -1$ (gradient),

$(x_1, y_1) = (1, 1)$.

$$y - 1 = -1(x - 1)$$

Simplify:

$$y - 1 = -x + 1$$

$$y = -x + 2$$

The equation of the tangent is

$$y = -x + 2$$