

Simplify expressions by collecting like terms

Simplify means answer in its simplest form.

If there are like terms in a mathematical expression, we must add (or subtract) them.

Like terms

like terms are algebraic terms with the same combination of variables.
This includes the exponents.

For example:

$(2x)$ and $(3x)$ are like terms because they have the variable (x) with the same power.

$(4x^2)$ and $(6x^2)$ are also like terms because they share the same variable and exponent.

However, $(2x)$ and $(3y)$ are not like terms (different letters), and $(4x^2)$ and $(6x^4)$ are not like terms (different powers).

Example

Simplify the following expressions:

a) $2a + 3b - a + 4b$

b) $(2a - a) + (3b + 4b)$

c) $4x^2 + 3x - 2x^2 + 5x$

d) $7m - 3n + 4m + 2n$

a)

$$\boxed{2a} + \boxed{3b} - \boxed{a} + \boxed{4b} = a + 7b$$

c)

$$\boxed{4x^2} + \boxed{3x} - \boxed{2x^2} + \boxed{5x} = 2x^2 + 8x$$

b)

$$\boxed{(2a - a)} + \boxed{(3b + 4b)} = a + 7b$$

d)

$$\boxed{7m} - \boxed{3n} + \boxed{4m} + \boxed{2n} = 11m - n$$

Distributive law

Distributive law: A term outside a bracket multiplies each of the terms inside the bracket.

Consider the following expressions,
we can **expand** the brackets using the distributive law :

$$2(3x + 5y) = 6x + 10y$$

$$3x(5 - 2x + x^2) = 15x - 6x^2 + 3x^3$$

Products two brackets:

$$(1 - x)(2 + x + x^2) = 2 \boxed{+x} \boxed{+x^2} \boxed{-2x} \boxed{-x^2} - x^3$$
$$= -x^3 - x + 2$$

We rewrite in descending order
(we add and subtract like terms)

Distributive law

Products of more than two brackets:

$$\begin{aligned}(x + 1)(2 - x)(3 + 2x) &= (2x - x^2 + 2 - x)(3 + 2x) \\ &= (x - x^2 + 2)(3 + 2x) \\ &= 3x + 2x^2 - 3x^2 - 2x^3 + 6 + 4x \\ &= -2x^3 - x^2 + 7x + 6\end{aligned}$$

We rewrite in descending order
(we add and subtract like terms)

Example

Simplify the following expressions:

a) $2(5x - 1)$

b) $3x(2x - 4)$

a)

$$2(5x - 1) = 10x - 2$$

b)

$$3x(2x - 4) = 6x^2 - 12x$$

Example

Simplify the following expressions:

a) $5x(1 - 3x + 4x^2)$

b) $-x(3x^2 - 2x + 1)$

a)

$$5x(1 - 3x + 4x^2) = 5x - 15x^2 + 20x^3$$

b)

$$-x(3x^2 - 2x + 1) = -3x^3 + 2x^2 - x$$

Example

Simplify the following expressions:

a) $(2 + x)(1 - 2x - x^2)$

b) $(2x - 1)(x^2 + 3x - 4)$

a)

$$\begin{aligned}
 (2 + x)(1 - 2x - x^2) &= 2 \boxed{-4x} \boxed{-2x^2} \boxed{+x} \boxed{-2x^2} - x^3 \\
 &= -x^3 - 4x^2 - 3x + 2
 \end{aligned}$$

We rewrite in descending order
(we add and subtract like terms)

b)

$$\begin{aligned}
 (2x - 1)(x^2 + 3x - 4) &= 2x^3 \boxed{+6x^2} \boxed{-8x} \boxed{-x^2} \boxed{-3x} + 4 \\
 &= 2x^3 + 5x^2 - 11x + 4
 \end{aligned}$$

We rewrite in descending order
(we add and subtract like terms)

Example

Simplify the following expression:

$$(2 + x)(x - 3)(1 + 3x)$$

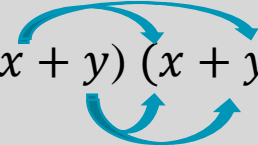
$$(2 + x)(x - 3)(1 + 3x) = (2x - 6 + x^2 - 3x)(1 + 3x) = (-x - 6 + x^2)(1 + 3x)$$

We rewrite in descending order
(we add and subtract like terms)

$$= -x - 3x^2 - 6 - 18x + x^2 + 3x^3$$
$$= 3x^3 - 2x^2 - 19x - 6$$

Perfect square expansion

Perfect squares expansion is a topic in Mathematics that involves expanding binomials raised to the power of 2. The perfect square formula for expanding binomials raised to power 2 is:

$$(x + y)^2 = (x + y)(x + y)$$

$$= x^2 + (x)(y) + (x)(y) + y^2$$

$$(x + y)^2 = (x)^2 + 2(x)(y) + (y)^2$$

Square the first term

Add twice the product of the first and last terms.

Add on the square of the last term.

For example: $(3 - 2x)^2 = (3)^2 + 2(3)(-2x) + (-2x)^2$

$$= 9 - 12x + 4x^2$$

Example

Simplify the following expressions:

a) $(2 + 5x)^2$

b) $(2x^2 - 7x)^2$

$$\begin{aligned} \text{a) } (2 + 5x)^2 &= (2)^2 + 2(2)(5x) + (5x)^2 \\ &= 4 + 20x + 25x^2 \end{aligned}$$

$$\begin{aligned} \text{b) } (2x^2 - 7x)^2 &= (2x^2)^2 + 2(2x^2)(-7x) + (-7x)^2 \\ &= 4x^4 - 28x^3 + 49x^2 \end{aligned}$$

Difference between two squares

The difference between two squares is another topic in Mathematics that involves factorising an algebraic expression. The difference between two squares formula is:

$$\begin{aligned}(x - y)(x + y) &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2\end{aligned}$$

$$(x - y)(x + y) = (x)^2 - (y)^2$$

Square the first term

Minus

Square of the last term

For example:

$$\begin{aligned}(3x - 5)(3x + 5) &= (3x)^2 - (5)^2 \\ &= 9x^2 - 25\end{aligned}$$

Example

Simplify the following expressions:

a) $(1 - x)(1 + x)$

b) $\left(\frac{2x}{3} + x^3\right)\left(\frac{2x}{3} - x^3\right)$

a)
$$(1 - x)(1 + x) = (1)^2 - (x)^2$$
$$= 1 - x^2$$

b)
$$\left(\frac{2x}{3} + x^3\right)\left(\frac{2x}{3} - x^3\right) = \left(\frac{2x}{3}\right)^2 - (x^3)^2$$
$$= \frac{4x^2}{9} - x^6$$

mathematicschapters.com

Simplify expressions by collecting like terms
and
Expansion

mathematicschapters.com