Complete the square

Completing the square is a method used in algebra to rewrite a quadratic expression in the form of $ax^2 + bx + c$ into a perfect square binomial. This method is especially useful for solving quadratic equations and converting them to vertex form.

$$ax^2 + bx + c = a(x+p)^2 + q \quad \text{where} \quad p = \frac{b}{2a} \text{ and } q = c - \frac{b^2}{4a}$$

Example

Rewrite the quadratic expression $x^2 - 2x - 4$ in the form $(x+p)^2 + q$.

$$ax^2 + bx + c = a(x+p)^2 + q \quad \text{where} \quad p = \frac{b}{2a} \quad \text{and} \quad q = c - \frac{b^2}{4a}$$

$x^2 - 2x - 4$

$a = 1$

$b = -2$

$c = -4$

$$\boldsymbol{p = \frac{b}{2a}}$$

$$= \frac{(-2)}{2(1)} = -1$$

$$\boldsymbol{q = c - \frac{b^2}{4a}}$$

$$= -4 - \frac{(-2)^2}{4(1)} = -5$$

$$\boldsymbol{ax^2 + bx + c = a(x+p)^2 + q}$$

$$x^2 - 2x - 4 = 1(x + (-1))^2 + (-5)$$

$$x^2 - 2x - 4 = (x-1)^2 - 5$$

Example

Rewrite the quadratic expression $2x^2+16x-7$ in the form $(x+p)^2+q$.

$2x^2+16x-7$

$a=2$

$b=16$

$c=-7$

$$\boldsymbol{p=\frac{b}{2a}}$$

$$=\frac{(16)}{2(2)}=4$$

$$\boldsymbol{q=c-\frac{b^2}{4a}}$$

$$=-7-\frac{(16)^2}{4(2)}$$

$$=-39$$

$$\boldsymbol{ax^2+bx+c=a(x+p)^2+q}$$

$$2x^2+16x-7=2(x+4)^2+(-39)$$

$$=2(x+4)^2-39$$

Example

Rewrite the quadratic expression $4x^2 - 2x + \frac{1}{4}$ in the form $(x+p)^2 + q$.

$$4x^2 - 2x + \frac{1}{4}$$

$a = 4$

$b = -2$

$c = \frac{1}{4}$

$$\boldsymbol{p = \frac{b}{2a}}$$

$$= \frac{(-2)}{2(4)} = \frac{-1}{4}$$

$$\boldsymbol{q = c - \frac{b^2}{4a}}$$

$$= \frac{1}{4} - \frac{(-2)^2}{4(4)}$$

$$= 0$$

$$\boldsymbol{ax^2 + bx + c = a(x+p)^2 + q}$$

$$4x^2 - 2x + \frac{1}{4} = 4(x + (\frac{-1}{4}))^2 + 0$$

$$= 4(x - \frac{1}{4})^2$$

Example

Rewrite the quadratic expression $-3x^2 + 3x + 7$ in the form $q - (x + p)^2$.

$$-3x^2 + 3x + 7$$

$a = -3$

$b = 3$

$c = 7$

$$\boldsymbol{p = \frac{b}{2a}}$$

$$= \frac{(3)}{2(-3)} = \frac{-1}{2}$$

$$\boldsymbol{q = c - \frac{b^2}{4a}}$$

$$= 7 - \frac{(3)^2}{4(-3)}$$

$$= \frac{31}{4}$$

$$\boldsymbol{ax^2 + bx + c = a(x - p)^2 + q}$$

$$-3x^2 + 3x + 7 = -3(x + (\frac{-1}{2}))^2 + \frac{31}{4}$$

$$= \frac{31}{4} - 3(x - \frac{1}{2})^2$$

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Complete the square

for expressions in the form $ax^2 + bx + c$

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