

Representing inequalities on a number line

Suppose our solution to an inequality is $x \geq k$, so every number that is k or greater than k is a possible value for x .

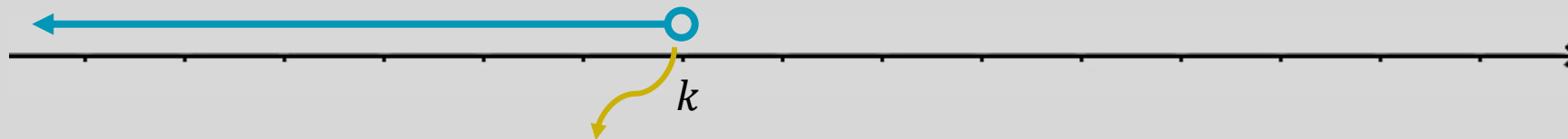
We could represent this on a number line by:



The filled-in circle indicates that k included.

The arrowhead indicates that all numbers on the number line, in this direction, is included.

Likewise, if our solution is $x < a$ our representation would be:



The hollow circle indicates that k is not included.

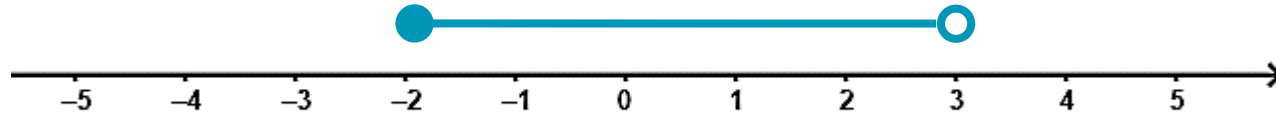
Example

Represent the following inequalities on a number line:

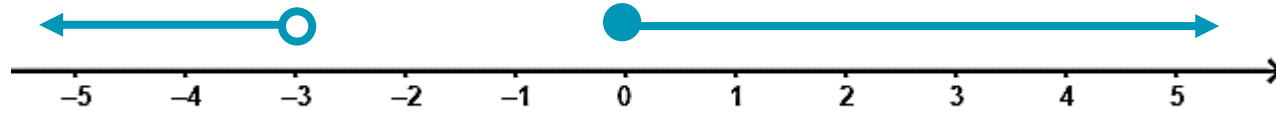
a) $-2 \leq x < 3$

b) $x < -3$ or $x \geq 0$

a) $-2 \leq x < 3$



b) $x < -3$ or $x \geq 0$



Linear inequalities

The process for solving linear inequalities is the same as that for solving linear equations, with the following exception:

Multiplying or dividing both sides by a negative number reverses the inequality sign.

For example:

$$\text{Solve for } x: \quad 5x - 2 \geq x - 4$$

$$5x - 2 - x \geq x - 4 - x$$

$$4x - 2 \geq -4$$

$$4x - 2 + 2 \geq -4 + 2$$

$$4x \geq -2$$

$$\frac{4x}{4} \geq \frac{-2}{4}$$

$$x \geq \frac{-1}{2}$$

On both sides, we have a variable.
We must remove the smaller variable.

Example

Solve for x and graph the solutions:

$$1 - 3x < 7$$

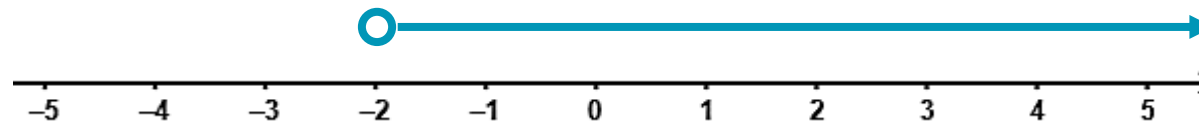
$$1 - 3x - 1 < 7 - 1$$

$$-3x < 6$$

$$\frac{-3x}{-3} < \frac{6}{-3}$$

$$x > -2$$

Multiplying or dividing both sides by a negative number reverses the inequality sign.



Example

Solve for x and graph the solutions :

$$2 - 3x \leq x - 1$$

$$2 - 3x \leq x - 1$$

$$2 - 3x + 3x \leq x - 1 + 3x$$

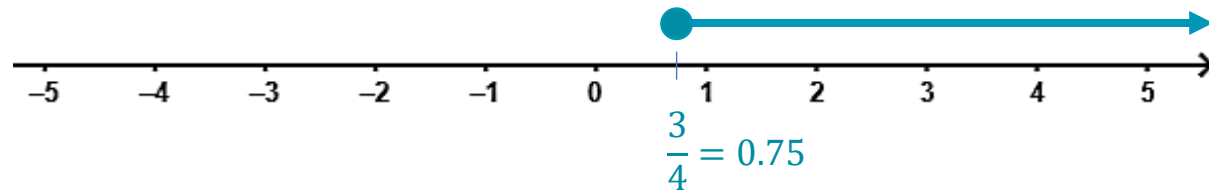
$$2 \leq 4x - 1$$

$$2 + 1 \leq 4x - 1 + 1$$

$$3 \leq 4x$$

$$\frac{3}{4} \leq \frac{4x}{4}$$

$$\frac{3}{4} \leq x$$



Example

Solve for x :

$$\frac{x - 3}{2} \leq x + 1$$

$$\frac{x - 3}{2} \leq x + 1$$

$$\cancel{2} \times \frac{x - 3}{\cancel{2}} \leq 2 \times (x + 1)$$

$$x - 3 - x \leq 2x + 2 - x$$

$$-3 \leq x + 2$$

$$-3 - 2 \leq x + 2 - 2$$

$$-5 \leq x$$

Example

Solve for x :

$$\frac{3-x}{2} > \frac{x+1}{3}$$

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$$\cancel{2} \times \frac{3-x}{\cancel{2}} > 2 \times \frac{x+1}{3}$$

$$3-x > \frac{2x+2}{3}$$

$$3 \times (3-x) > \cancel{3} \times \frac{2x+2}{\cancel{3}}$$

$$9-3x > 2x+2$$

$$9-3x+3x > 2x+2+3x$$

$$9 > 5x+2$$

$$9-2 > 5x+2-2$$

$$7 > 5x \quad \rightarrow \quad \frac{7}{5} > x$$

Example

Solve for x :

$$5x + \frac{2}{3} \geq 1$$

$$5x + \frac{2}{3} \geq 1$$

$$5x + \frac{2}{3} - \frac{2}{3} \geq 1 - \frac{2}{3}$$

$$5x \geq \frac{1}{3}$$

$$\frac{5x}{5} \geq \frac{1}{3} \times \frac{1}{5}$$

$$x \geq \frac{1}{3} \times \frac{1}{5}$$

$$x \geq \frac{1}{15}$$

Example

Solve for x and graph the solutions:

$$-3 \leq 2 - 5x < 5$$

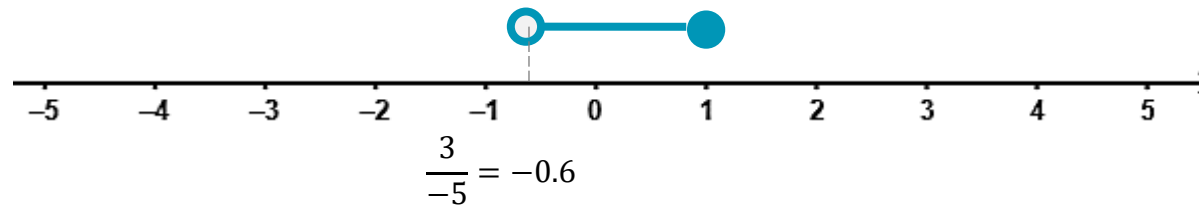
$$-3 - 2 \leq 2 - 5x - 2 < 5 - 2$$

$$-5 \leq -5x < 3$$

$$\frac{-5}{-5} \leq \frac{-5x}{-5} < \frac{3}{-5}$$

$$1 \geq x > \frac{3}{-5}$$

Multiplying or dividing both sides by a negative number reverses the inequality sign.



Example

Solve for x and graph the solutions:

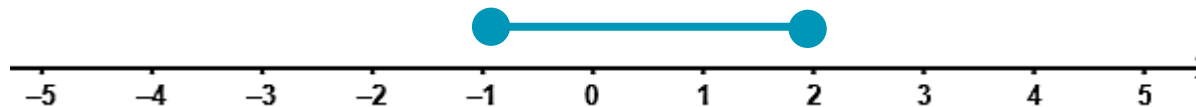
$$-1 \leq 2 + 3x \leq 8$$

$$-1 - 2 \leq 2 + 3x - 2 \leq 8 - 2$$

$$-3 \leq 3x \leq 6$$

$$\frac{-3}{3} \leq \frac{3x}{3} \leq \frac{6}{3}$$

$$-1 \leq x \leq 2$$



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including on a number line

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