Exponential growth

If an amount grows by a constant percentage per year, this eventually adds up to what we call exponential growth.

$$A = A_0(1 + r)^t$$

A is the Future amount, $\mathbf{A_0}$ is the Initial amount, r is the growth rate (estimation), and **t** is the Period.

Note:

- $\boldsymbol{Growth\ rate = \frac{New\ amount - original\ amount}{Original\ population}}$

Exponential growth

Exponential decay

If an amount decays by a constant percentage per year, this eventually adds up to what we call exponential decay.

$$A = A_0(1 - r)^t$$

A is the Future amount, $\mathbf{A_0}$ is the Initial amount, r is the decay rate (estimation), and **t** is the Period.

Note:

- $\boldsymbol{Decay\,rate = \frac{Original\ amount - New\ amount}{Original\ population}}$
- **In decay, the quantity approaches but never reaches zero.**

Example

A town's population is 10,000 and grows by 2% annually.

What will it be after 5 years?

$$A = A_0(1+r)^t$$

$$A = 10{,}000\left(1+\frac{2}{100}\right)^5$$

$$= 11040.81$$

$= 11041$ rounded to 11,041 **people**

Example

The population of one variety of butterfly is decreasing exponentially at a rate of 34% per year. At the end of 2014, the population was 125.9 million.

Calculate the population at the end of 2019.

$$\boldsymbol{A = A_0(1 - r)^t}$$

$$A = 125.9\left(1 - \frac{34}{100}\right)^5$$

$$= 15.77 \ \ million$$

Example

A radioactive substance decays by 12% each year.

How much of a 100g sample remains after 3 years?

$$\boldsymbol{A = A_0(1 - r)^t}$$

$$A = 100\left(1 - \frac{12}{100}\right)^3$$

$$= \frac{42592}{625}$$

$$= 68.15\ g$$

Half-life

Half-life is an important concept in mathematics and science, particularly in the study of radioactive decay.
It's the time required for a quantity to reduce to half its initial value.

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

A is the remaining amount, **A_0** is the initial amount, **t** is time, and **$t_{1/2}$** is the half-life

For example:

A radioactive isotope has a half-life of 5 days.
If you start with 100 grams of the isotope, how much will remain after 15 days?

$$A = 100 \left(\frac{1}{2}\right)^{\frac{15}{5}}$$

$$= 12.5\ g$$

Example

A medication has a half-life of 4 hours in the human body.

If a patient takes a 200 mg dose, how much of the drug remains in their system after 12 hours?

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$A = 200 \left(\frac{1}{2}\right)^{\frac{12}{4}}$$

$$= 25\ g$$

mathematicschapters.com