Ratios

A ratio is a way of comparing two or more quantities.

For example: There are 4 rectangles to 1 circle.

Ratios are typically written with a colon 4 : 1 meaning 4 to 1.

To write the ratio of two parameters, first make their units the same, then write the ratio and remove the units

Note: A ratio itself does not have a unit because it is a comparison of two quantities of the same kind. The units cancel out when forming the ratio, leaving a pure number.

For example: Ashton has \$5 and Ashley has 50 cents. Write as a ratio, without simplifying your answer.

$\$5 \times 100 = 500\ cents$

$500\ cent : 50\ cent$

$500 : 50$

Example

A bag contains 3 red balls, 4 blue balls, and 5 green balls.

What is the ratio of red balls to blue balls to green balls?

The ratio of red balls to blue balls to green balls is $3:4:5$

Example

A bag contains 3 red balls, 4 blue balls, and 5 green balls.

What is the fraction of red balls?
What is the fraction of blue balls?
What is the fraction of green balls?

The ratio of red balls to blue balls to green balls is 3:4:5

The total number of parts in the ratio is 3 + 4 + 5 = 12. Therefore:

The fraction of red balls is $\frac{3}{12} = \frac{1}{4}$

The fraction of blue balls is $\frac{4}{12} = \frac{1}{3}$

The fraction of green balls is $\frac{5}{12}$

Example

Noah has 25 colour balls and Olivia has 32 colour balls.

Find the fraction $\frac{\text{Olivia's colour balls}}{\text{Noah's colour balls}}$ in the simplest form.

$$\frac{\text{Olivia's colour balls}}{\text{Noah's colour balls}} = \frac{32}{25}$$

Simplifying Ratios

You can simplify ratios by dividing the numbers by common factors, just like you do with fractions.

For example: There are 30 mathematics books and 20 physics books on a shelf.
Write down the ratio of mathematics books to physics books in its simplest form.

mathematics books:physics books → 30 books : 20 books → 30 : 20 → 3 : 2

$$\frac{30}{20} = \frac{3}{2}$$

The ratio of mathematics books to physics books is 3: 2.

Example

Write the ratio 2m to 60 cm in its simplest form.

$$\frac{200}{60} = \frac{10}{3}$$

$2m\ to\ 60cm \rightarrow 200cm : 60cm \rightarrow 200 : 60 \rightarrow 10:3$

Rewrite the ratio so that the units are the same

Remove the units altogether

Simplify as normal

The ratio $2m\ to\ 60\ cm \rightarrow 10:3$

Using ratio to divide quantities

To divide a quantity in a given ratio, you can use the unitary method or the fraction method.

To divide a value by a given ratio, proceed in the following steps:

1. Find the total number of parts in the ratio by adding the values of the ratio.
2. Divide the number of each share by the total number of parts.
3. Multiply the quantity by the corresponding ratio to find each share.

Example

Divide \$50 in the ratio $2 : 3$.

1. Find the total number of parts in the ratio by adding the values of the ratio.

There are $2 + 3 = 5$ parts.

2. Divide the number of each share by the total number of parts.

The ratio $2 : 3$. $\frac{2}{5}$ $\frac{3}{5}$

3. Multiply the quantity by the corresponding ratio to find each share.

$50 \times \frac{2}{5} = \$\,20$ $50 \times \frac{3}{5} = \$\,30$

Example

Divide \$200 in the ratio $1:3:4$.

There are $1+3+4=8$ parts.

the ratio $1:3:4$ $\quad \frac{1}{8} \quad \frac{3}{8} \quad \frac{4}{8}$

$200 \times \frac{1}{8} = 25$ $\qquad$ $200 \times \frac{3}{8} = 75$ $\qquad$ $200 \times \frac{4}{8} = 100$

Example

The ratio of boys to girls in a class is 5:3. If there are 40 students in the class, how many boys and girls are there?

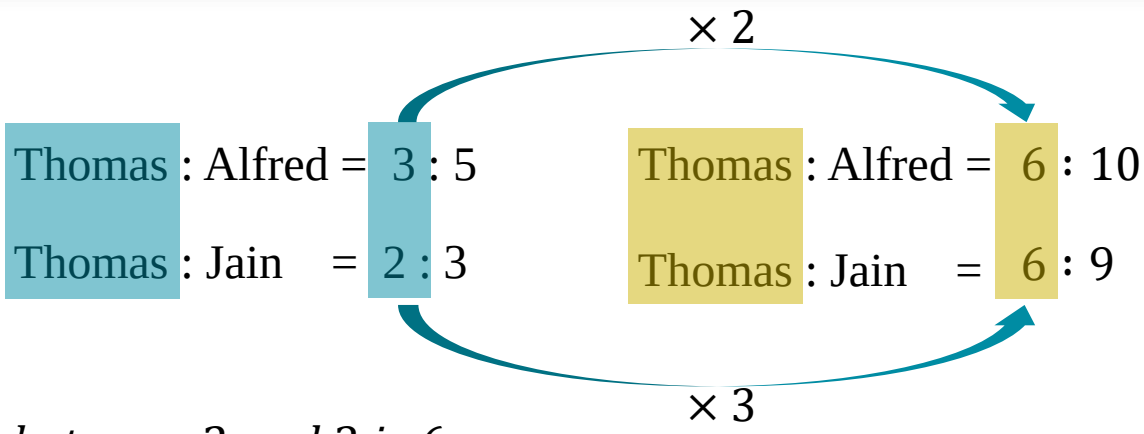
$5 + 3 = 8$

5:3 $\quad Boys\ \frac{5}{8} \quad Girls\ \frac{3}{8}$

$There\ are\ \frac{5}{8} \times 40 = 25\ boys \qquad There\ are\ \frac{3}{8} \times 40 = 15\ girls$

Example

Thomas, Alfred and Jain share 200$.
The ratio Thomas : Alfred = 3:5.
The ratio Thomas : Jain = 2:3.
Calculate how much each person received.



Lcm between 2 and 3 is 6

Thomas : Alfred : Jain = $6 : 10 : 9$

$6 + 10 + 9 = 25$

$\frac{6}{25} \quad \frac{10}{25} \quad \frac{9}{25}$

Thomas = $200 \times \frac{6}{25} = 48\ \$$

Alfred = $200 \times \frac{10}{25} = 80\ \$$

Jain = $200 \times \frac{9}{25} = 72\ \$$

Map scales

A representation of a real thing with correct sizes reduced or enlarged by a certain amount is called the scale.

Scale Ratio:

The scale is usually given as a ratio, such as 1:50,000. This means that 1 unit on the map (for example 1 cm) represents 50,000 of the same units in real life (50,000 cm or 500 meters).

The scale for a drawing is the ratio between the size of the drawing and what is represented.

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{1}{n}$$

$$\frac{Area\ of\ drawing}{Real\ area} = \frac{1}{n^2}$$

Example

In a map the scale is 5 cm represented 200 m.

Find the scale in the form $1:n$.

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{1}{n}$$

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{5\ cm}{200 \times 100\ cm}$$

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{5}{20000}$$

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{\frac{5}{5}}{\frac{20000}{5}}$$

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{1}{4000}$$

Example

A map is drawn to a scale of 1:100000. Find the actual distance represented by 5cm on the map. Give you answer in metres.

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{1}{n}$$

$$\frac{5\ cm}{Real\ length} = \frac{1}{100000}$$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction

$$500000\ cm = Real\ length$$

$$\frac{m}{500000} = \frac{1}{100} \rightarrow m = \frac{500000}{100}$$

m : cm = 1:100 **or** $\frac{m}{cm} = \frac{1}{100}$

$$Real\ length = 5000\ m$$

Example

The rectangular garden on the 1:1000 scale plan is 20 cm and 15 cm.
The perimeter of the garden is? Give you answer in metres.

$$\frac{Length\ of\ drawing}{Real\ length} = \frac{1}{n}$$

$$\frac{20\ cm}{Real\ length} = \frac{1}{1000}$$

$$20000\ cm = Real\ length$$

$$\frac{m}{20000} = \frac{1}{100} \rightarrow length = 200m$$

$$\frac{15\ cm}{Real\ \ width} = \frac{1}{1000}$$

$$15000\ cm = Real\ width$$

$$\frac{m}{15000} = \frac{1}{100} \rightarrow width = 150m$$

m : cm = 1:100
or $\frac{m}{cm} = \frac{1}{100}$

Perimeter of Rectangle

$$P = 2(l + w)$$

$$= 2(200 + 150)$$

$$= 700\ m$$

Example

The square garden has an area of 36 square meters.
What area will its image have in the 1:60 scale? Give you answer in cm^2 .

$$\frac{Area\ of\ drawing}{Real\ area} = \frac{1}{n^2}$$

$$\frac{Area\ of\ drawing}{36\ m^2} = \frac{1}{60^2}$$

$$Area\ of\ drawing = \frac{36\ m^2}{60^{\ 2}} = 0.01\ m^2$$

$$\frac{m^2}{cm^2} = \frac{1}{10000} \quad \rightarrow \quad 0.01 \times 10000 = 100 cm^2$$

$$Area\ of\ drawing = 100\ cm^2$$

m : cm = 1:100

or $\frac{m}{cm} = \frac{1}{100}$

Therefore $(\frac{m}{cm})^2 = (\frac{1}{100})^2$

or $\frac{m^2}{cm^2} = \frac{1}{10000}$

Proportion

A proportion is a statement that two ratios are equal. (Proportion is an equation that determines that the given ratios are equivalent or corresponding to one another, that is, the proportion expresses the equality of ratios/ fractions.)

At this level, we study two types of proportions.

1) Direct Proportion.
2) Inverse Proportion.

Direct proportion

Two quantities are directly proportional when an increase in one quantity causes an increase in the other and vice versa (a decrease in one quantity causes a decrease in the other). $a \propto b$ (a proportional to b)

Proportionality can be used to set up an equation. $a \propto b \rightarrow a = k \times b$

There are four steps to do this:

1. write the proportional relationship
2. convert to an equation using a constant of proportionality
3. use given information to find the constant of proportionality
4. substitute the constant of proportionality into the equation

constant of proportionality

For example:

A recipe for pancakes requires 2 cups of flour for every 3 cups of milk. calculate, for 12 cups of flour, how much milk do you need?

In this recipe, by increasing the number of cups of flour, the number of cups of milk should also increase, therefore this proportioning is of a direct type.

1. $cups\ of\ flour \propto cups\ of\ milk$
2. $cups\ of\ flour = \boldsymbol{k} \times cups\ of\ milk$
3. $2 = k \times 3 \rightarrow \frac{2}{3} = k$
4. $cups\ of\ flour = \frac{2}{3} \times cups\ of\ milk \qquad 12 = \frac{2}{3} \times cups\ of\ milk \qquad 18 = cups\ of\ milk$

Example

The ratio of Ashton's height to Ashley's height is $7:6$. If Ashton is $1.63\ m$ tall, how tall is Ashley?

Two quantities are directly proportional when an increase in one quantity causes an increase in the other and vice versa (a decrease in one quantity causes a decrease in the other).

$\text{Ashton} \propto \text{Ashley}$

$\text{Ashton} = k \times \text{Ashley}$

$7 = k \times 6 \quad \rightarrow \quad \frac{7}{6} = k$

$\text{Ashton} = \frac{7}{6} \times \text{Ashley}$

$1.63 = \frac{7}{6} \times \text{Ashley}$

$\frac{6 \times 1.63}{7} = \text{Ashley}$

$1.40\ \text{m} = \text{Ashley}$

Example

Tahani is making lemonade juice for a party. She mixed 4 liters of water for every ten pieces of lemonade. How many liters of water does she need if she has 28 lemonades?

Two quantities are directly proportional when an increase in one quantity causes an increase in the other and vice versa (a decrease in one quantity causes a decrease in the other).

liters of water $\propto$ pieces of lemonade

liters of water $= k \times$ pieces of lemonade

$4 = k \times 10 \quad \rightarrow \quad \frac{4}{10} = k$

liters of water $= \frac{4}{10} \times$ pieces of lemonade

liters of water $= \frac{4}{10} \times$ pieces of lemonade

liters of water $= \frac{4}{10} \times 28$

liters of water $= 11.2$

Inverse proportion

Two quantities are inversely proportional when an increase in one causes a decrease in the other and vice versa (a decrease in one causes an increase in the other). $a \propto \frac{1}{b}$ (a, inversely proportional to b)

Proportionality can be used to set up an equation. $a \propto \frac{1}{b} \rightarrow a = \frac{k}{b}$ (k: constant of proportionality)

There are four steps to do this:

1. write the proportional relationship
2. convert to an equation using a constant of proportionality
3. use given information to find the constant of proportionality
4. substitute the constant of proportionality into the equation

For example: y inversely proportion to x. When $x = 9, y = 8$. Find y when $x = 6$.

1. $y \propto \frac{1}{x}$
2. $y = \frac{k}{x}$
3. $8 = \frac{k}{9} \rightarrow k = 72$
4. $y = \frac{72}{x}$ $y = \frac{72}{6} \rightarrow y = 12$

Example

3 workers worked 18 days to paint a building.

If the number of workers was 6, how many days would this work be done?

Two quantities are inversely proportional when an increase in one causes a decrease in the other and vice versa (a decrease in one causes an increase in the other).

In this matter, with the increase of workers, the working time decreases.

$$w \propto \frac{1}{d}$$

$$w = \frac{k}{d}$$

$$3 = \frac{k}{18} \quad \rightarrow \quad k = 54$$

$$w = \frac{54}{d}$$

$$6 = \frac{54}{d} \quad \rightarrow \quad d = \frac{54}{6} = 9$$

Example

The time taken to complete a journey is inversely proportional to the speed. If it takes 2 hours to complete the journey at a speed of 60 km/h, how long will it take to complete the journey at a speed of 80 km/h?

Two quantities are inversely proportional when an increase in one causes a decrease in the other and vice versa (a decrease in one causes an increase in the other).

$$h \propto \frac{1}{v}$$

$$h = \frac{k}{v}$$

$$2 = \frac{k}{60} \quad \rightarrow \quad k = 120$$

$$h = \frac{120}{v}$$

$$h = \frac{120}{80} \quad \rightarrow h = \frac{3}{2} = 1.5$$

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