Upper and lower bounds

Upper and lower bounds are used to determine accuracy.

The **upper bounds** are the maximum values that a number could have been before it was rounded.

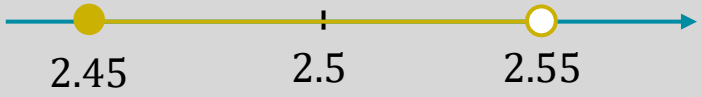
The **lower bounds** are the minimum values that a number could have been before it was rounded.

They are written in the form of inequalities.

For example:

2.5 is rounded to one decimal place.

Any number from 2.45 up to but not including 2.55 would be rounded to 2.5.

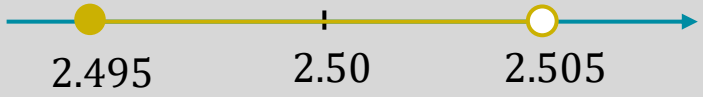


$2.45 \leq x < 2.55$

lower bound — Upper bound

2.50 is rounded to two decimal place.

Any number from 2.495 up to but not including 2.505 would be rounded to 2.50.



$2.495 \leq x < 2.505$

lower bound — Upper bound

In the following, you will learn how to calculate the upper and lower bounds

limits of accuracy

Upper and lower bounds can also be called limits of accuracy.
The upper and lower bounds can be written using error intervals

The limit of accuracy of a measurement is the range of possible values a measurement can be when a measurement is given.

The **lower limit** is the smallest value that would round up to the estimated value.
The **upper limit** is the smallest value that would round up to the next estimated value.

Error intervals show the range of numbers that are within the limits of accuracy.

Find limits of accuracy (upper and lower bounds)

1. Identify the place value of the degree of accuracy stated.
2. Divide this place value by 2.
3. Add this amount to the given value to find the upper bound, subtract this amount from the given value to find the lower bound.

Error intervals: $Lower\ bound \leq y < Upper\ bound$

For example:

A mass of 3 kg orange to the nearest kilogram. What are the upper and lower bounds for the weights of the oranges?

Step 1 The degree of accuracy is to the nearest **1 *Kg***.

Step 2 $1 \div 2 = 0.5\ Kg$

Step 3 Upper limit = $3 + 0.5 = 3.5$

Lower limit = $3 - 0.5 = 2.5$

Error intervals: $2.5 \leq weight < 3.5$

Lower bound or lower limit

Upper bound or upper limit

Example

The population of a town, y, is given as 280000 people rounded to the nearest 10000.

Find the upper and lower bounds of y and state the error interval.

The place value of the degree of accuracy is 10000.

$10000 \div 2 = 5000$

$280000 + 5000 = 285000$ Upper bound

$280000 - 5000 = 275000$ Lower bound

Error interval: $275000 \leq y < 285000$

Lower bound or lower limit

Upper bound or upper limit

Example

The weight, w of a parcel is recorded as 440g to the nearest 10g.

Write down the error interval for the weight of the parcel.

$10 \div 2 = 5$

$440 - 5 = 435$ Lower bound or lower limit

$440 + 5 = 445$ Upper bound or upper limit

The error interval of the weight: $435 \leq \text{weight} < 445$

Calculations Using Bounds

When performing calculations involving bounds:

Operation	Upper bound	Lower bound
Addition	Upper bound + Upper bound	Lower bound + Lower bound
Subtraction	Upper bound − Lower bound	Lower bound − Upper bound
Multiplication	Upper bound × Upper bound	Lower bound × Lower bound
Division	Upper bound ÷ Lower bound	Lower bound ÷ Upper bound

Example

Suppose two values are 10 cm and 5 cm, both rounded to the nearest cm.

Calculate:

a) Upper bound of sum.

b) Lower bound of sum.

c) Upper bound of difference.

d) Lower bound of difference.

Nearest cm 1 cm $1 \div 2 = 0.5$

For 10 cm: lower bound: $10 - 0.5 = 9.5\ cm$, upper bound: $10 + 0.5 = 10.5\ cm$.

For 5 cm: lower bound: $5 - 0.5 = 4.5\ cm$, upper bound: $5 + 0.5 = 5.5\ cm$.

a) Upper bound of sum

Addition:
Upper bound = Upper bound + Upper bound

Upper bound of sum: $10.5 + 5.5 = 16$

b) Lower bound of sum

Addition:
Upper bound = Upper bound + Lower bound

Lower bound of sum: $9.5 + 4.5 = 14$

c) Upper bound of difference

Subtraction:
Upper bound = Upper bound - Lower bound

Upper bound of difference: $10.5 - 4.5 = 6$

d) Lower bound of difference

Subtraction:
Lower bound = Lower bound − Upper bound

Lower bound of difference: $9.5 - 5.5 = 4$

Example

Suppose two mass values are 200 g and 100 g, both rounded to the nearest gram.

Calculate:

a) Upper bound of product.

b) Lower bound of product.

c) Upper bound of divide.

d) Lower bound of divide.

Nearest gram 1 g $1 \div 2 = 0.5$

For 200 cm: lower bound: $200 - 0.5 = 199.5\ cm$, upper bound: $200 + 0.5 = 200.5\ cm$.

For 100 cm: lower bound: $100 - 0.5 = 99.5\ cm$, upper bound: $100 + 0.5 = 100.5\ cm$.

a) Upper bound of product.

Multiplication:
Upper bound = Upper bound × Upper bound

Upper bound of product: $200.5 \times 100.5 = 20150.25$

b) Lower bound of product.

Multiplication:
Upper bound = Lower bound × Lower bound

Lower bound of product: $199.5 \times 99.5 = 19850.25$

c) Upper bound of divide.

Division:
Upper bound = Upper bound ÷ Lower bound

Upper bound of divide: $200.5 \div 99.5 = 2.0157$

d) Lower bound of divide.

Division:
Lower bound = Lower bound ÷ Upper bound

Lower bound of divide: $199.5 \div 100.5 = 1.9850$

Example

A rectangle has a width W of 3 cm to the nearest centimetre and a length L of 8.6 cm to the nearest millimetre.
Find the upper and lower bounds of the perimeter, P, of the rectangle.

$1 \div 2 = 0.5$

$3 + 0.5 = 3.5$

$3 - 0.5 = 2.5$

The error interval of the width: $2.5 \leq \text{width} < 3.5$

Rectangle $perimeter = 2(\text{width}+\text{length})$

Operation	Upper bound	Lower bound
Addition	Upper bound + Upper bound	Lower bound + Lower bound

$$2(2.5 + 8.55) \leq \text{perimeter} < 2(3.5 + 8.65)$$

$$22.1\ cm \leq \text{perimeter} < 24.3\ cm$$

Lower bound or lower limit

Upper bound or upper limit

$1\ millimetre = 0.1\ centimetre$

$0.1 \div 2 = 0.05$

$8.6 + 0.05 = 8.65$

$8.6 - 0.05 = 8.55$

The error interval of the length: $8.55 \leq \text{length} < 8.65$

Example

A *parallelogram* has a base b of 5.64 m to 2 decimal places and a perpendicular height h of 2.3 m to 2 significant figures.
Find the upper and lower bounds of the area A of the parallelogram.

$0.01 \div 2 = 0.005$

$5.64 + 0.005 = 5.645$

$5.64 + 0.005 = 5.635$

The error interval of the base: $5.635 \leq \text{base} < 5.645$

Parallelogram area $= base \times height$

Operation	Upper bound	Lower bound
Multiplication	Upper bound × Upper bound	Lower bound × Lower bound

$5.635 \times 2.25 \leq area < 5.645 \times 2.35$

$12.67875\ m^2 \leq area < 13.26575\ m^2$

Lower bound or lower limit

Upper bound or upper limit

2.3 m to 2 significant figures

$0.1 \div 2 = 0.05$

$2.3 + 0.05 = 2.35$

$2.3 - 0.05 = 2.25$

The error interval of the height: $2.25 \leq \text{height} < 2.35$

Example

Alfred has a piece of wood of length 43cm, correct to the nearest cm. From this ribbon he cuts off a piece with length 67mm, correct to the nearest mm.
Work out the lower bound and the upper bound for the length of the remaining wood.
Give your answer in centimetres.

$1 \div 2 = 0.5$

$43 + 0.5 = 43.5\ cm$ Upper bound

$43 - 0.5 = 42.5\ cm$ lower bound

Operation	Upper bound	Lower bound
Subtraction	Upper bound – Lower bound	Lower bound – Upper bound

$43.5 - 6.65 = 36.85$ Upper bound

$42.5 - 6.75 = 35.75$ Lower bound

$1\ millimetre = 0.1\ centimetre$

$67\ mm \times 0.1 = 6.7\ cm$

$0.1 \div 2 = 0.05$

$6.7 + 0.05 = 6.75$ Upper bound

$6.7 - 0.05 = 6.65$ Lower bound

Example

A car travel 550 km, correct to the nearest 10 km.
The time taken to completed the journey is 7 hours, correct to the nearest half hours.

Calculate the lower bound and upper bound of the speed of the car.

$10 \div 2 = 5$

$550 + 5 = 555$ Upper bound

$550 - 5 = 545$ Lower bound

$0.5 \div 2 = 0.25$

$7 + 0.25 = 7.25$ Upper bound

$7 - 0.25 = 6.75$ Lower bound

$$speed = \frac{distance}{time}$$

Operation	Upper bound	Lower bound
Division	Upper bound ÷ Lower bound	Lower bound ÷ Upper bound

$$speed = \frac{555}{6.75}$$

$= 82.22\ km/h$ Upper bound

$$speed = \frac{545}{7.25}$$

$= 75.17\ km/h$ Lower bound

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