Similar Triangles

Two triangles will be similar if the angles are equal (corresponding angles), and sides are in the same ratio or proportion(corresponding sides).

The symbol used to express similar shapes is “∼”

Similar Triangles Theorems

- **AA (or AAA) or Angle-Angle Similarity**

  AA similarity criterion states that if any two angles in a triangle are equal to any two angles of another triangle, they must be similar triangles.

- **SSS or Side-Side-Side Similarity**

  According to the SSS similarity theorem, two triangles will be similar if the corresponding ratio of all the sides of the two triangles is equal.

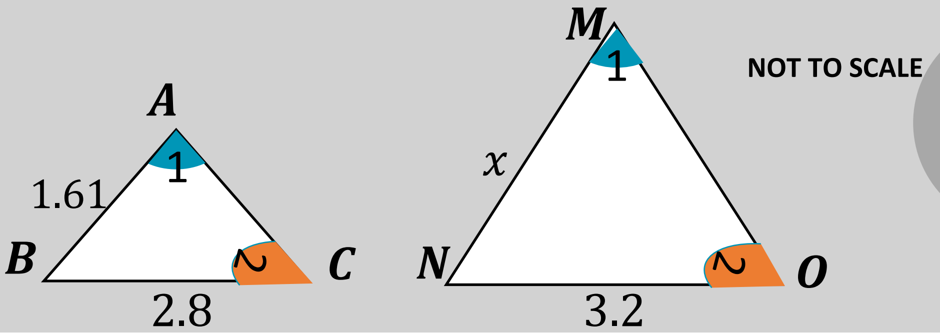
- **SAS or Side-Angle-Side Similarity**

  If the two sides of a triangle are in the same proportion as the two sides of another triangle, and the angles inscribed by the two sides in both triangles are equal, then two triangles are said to be similar.

Example

Triangle ABC is similar to triangle MNO.

Find the value of x.



	1	2	3
Small	2.8	1.61	AC
Large	3.2	x	MO

$$\frac{2.8}{3.2} = \frac{1.61}{x}$$

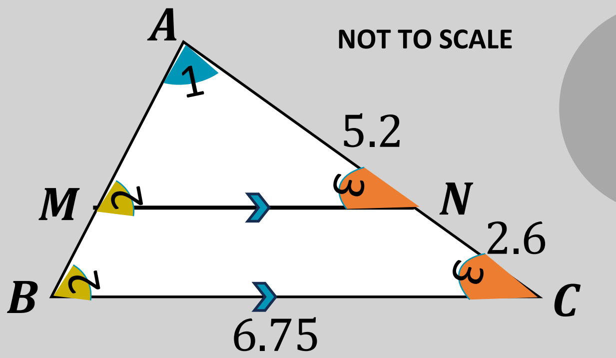
$$\frac{3.2 \times 1.61}{2.8} = x$$

$$1.84 = x$$

Example

MN is parallel to BC.

Calculate the length of MN.



MN is parallel to BC $\Rightarrow$

Angle AMN is equal to angle ABC
(Corresponding angles)

Angle ANC is equal to angle ACB
(Corresponding angles)

- **AA (or AAA) or Angle-Angle Similarity**

Triangle ABC similar to triangle AMN

	1	2	3
Small	MN	AN	AM
Large	BC	AC	AB

$$\boxed{\frac{MN}{6.75} = \frac{5.2}{5.2 + 2.6}} = \frac{AM}{AB}$$

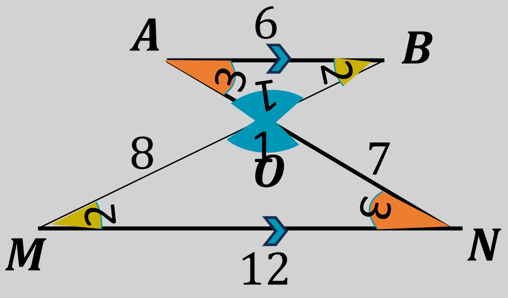
$$MN = \frac{6.75 \times 5.2}{5.2 + 2.6}$$

$$MN = 4.5$$

Example

In the diagram AB is parallel to MN.

Calculate the length of OB.



NOT TO SCALE

MN is parallel to AB ⇨

Angle BMN is equal to angle ABM
(Alternate angles)

Angle ANM is equal to angle BAN
(Alternate angles)

- **AA (or AAA) or Angle-Angle Similarity**

$Triangle\ ABO\ similar\ to\ triangle\ MNO$

	1	2	3
Small	AB	OA	OB
Large	MN	ON	OM

$$\frac{6}{12} = \frac{OA}{ON} = \frac{OB}{8}$$

$$\frac{6}{12} = \frac{OB}{8}$$

$$\frac{6 \times 8}{12} = OB$$

$$4 = OB$$

Similar shapes

The ratio of corresponding sides of similar shapes = **scale factor** *or* k

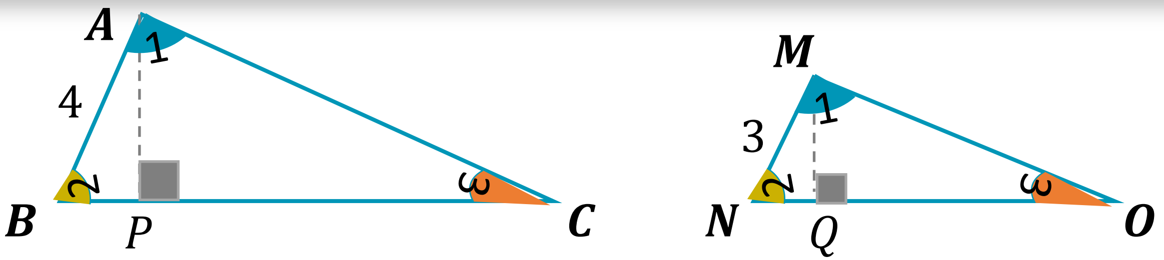
The ratio of corresponding heights of similar shapes = **scale factor** *or* k

The ratio of perimeters of similar shapes = **scale factor** *or* k

The ratio of areas of similar shapes = **(scale factor)²** *or* k^2

The ratio of volumes of similar shapes = **(scale factor)³** *or* k^3

For example:



	1	2	3
Small	ON	OM	MN
Large	BC	AC	AB

$$\frac{ON}{BC} = \frac{OM}{AC} = \frac{3}{4} = k$$

The ratio of heights $\frac{MN}{AP} = k = \frac{3}{4}$

The ratio of Perimeters $\frac{P_{MNO}}{P_{ABC}} = k = \frac{3}{4}$

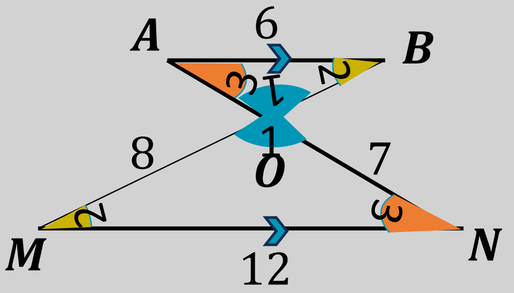
The ratio of Areas $\frac{S_{MNO}}{SABC} = k^2 = (\frac{3}{4})^2 = \frac{9}{16}$

C´

Example

In the diagram AB is parallel to MN.
The area of triangle OMN is 26.906 cm^2 .

Calculate the area of OAB.



NOT TO SCALE

MN is parallel to AB ⇒

Angle BMN is equal to angle ABM
(Alternate angles)

Angle ANM is equal to angle BAN
(Alternate angles)

- **AA (or AAA) or Angle-Angle Similarity**

Triangle OAB similar to triangle OMN

	1	2	3
Small	AB	OA	OB
Large	MN	ON	ON

$$\frac{6}{12} = \frac{OA}{ON} = \frac{OB}{8} = k$$

$$\frac{6}{12} = k \qquad \frac{1}{2} = k$$

The ratio of areas of similar shapes = (scale factor)^2

$$\frac{smaller\ area}{larger\ area} = k^2$$

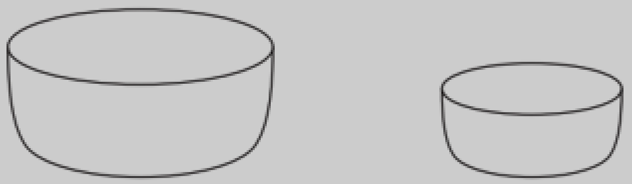
$$\frac{S_{OAB}}{S_{OMN}} = (\frac{1}{2})^2$$

$$\frac{S_{OAB}}{26.906} = \frac{1}{4} \qquad S_{OAB} = \frac{26.906 \times 1}{4} \qquad S_{OAB} = 6.73$$

Example

The diagram shows two similar bowls.
The larger bowls has capacity 7.8 litres,
and the smaller bowls has capacity 4 litres.
The larger bowls has height 11.5 cm.

Find the height of the smaller bowl.



The ratio of volumes of similar shapes = (scale factor)^3

$$\frac{smaller\ capacity}{larger\ capacity} = k^3 \qquad \frac{4}{7.8} = k^3 \implies \sqrt[3]{\frac{4}{7.8}} = k$$

The ratio of corresponding heights of similar shapes = scale factor

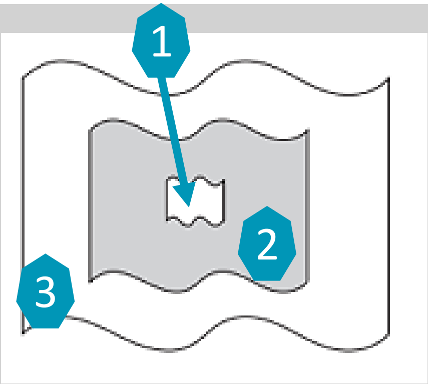
$$\frac{smaller\ height}{larger\ height} = k \qquad \frac{x}{11.5} = \sqrt[3]{\frac{4}{7.8}} \qquad x = 11.5 \times \sqrt[3]{\frac{4}{7.8}}$$

$$x = 9.20$$

Example

The diagram shows three shapes that are similar.
The height of the shapes are in ratio 1:5:8.

Calculate the ratio shaded area: total unshaded area.
Give your answer in simplest form.



The ratio of areas of similar shapes = (scale factor)^2

$$\frac{Area\ 1}{Area\ 2} = (\frac{1}{5})^2 \qquad Area\ 1 = \frac{1}{25}\ Area\ 2$$

$$\frac{Area\ 2}{Area\ 3} = (\frac{5}{8})^2$$

$$Area\ 3 = \frac{64}{25}\ Area\ 2$$

The shaded area = Area 2 - Area 1

$$= \text{Area 2} - \frac{1}{25}\ Area\ 2 = \frac{24}{25}\ Area\ 2$$

The total unshaded area = Area 3 - Area 2 + Area 1

$$= \frac{64}{25}\ Area\ 2 - Area\ 2 + \frac{1}{25}\ Area\ 2$$

$$= \frac{40}{25}\ Area\ 2$$

The ratio shaded area: total unshaded area

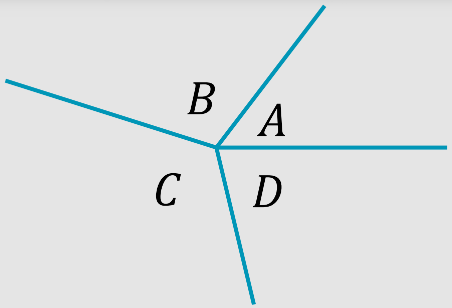
$$\frac{\frac{24}{25}\ \cancel{Area\ 2}}{\frac{40}{25}\ \cancel{Area\ 2}} = \frac{3}{5} \qquad \text{or} \qquad 3:5$$

Angle properties

Angles at a point

The sum of the sizes of the angles at a point is 360°

$$A + B + C + D = 360$$



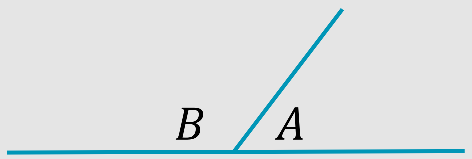
Angle properties

Angles at a point

Angles at a point on a straight line

The sum of the sizes of the angles on a line is 180º. The angles are supplementary.

$$A + B = 180$$



Angle properties

Angles at a point

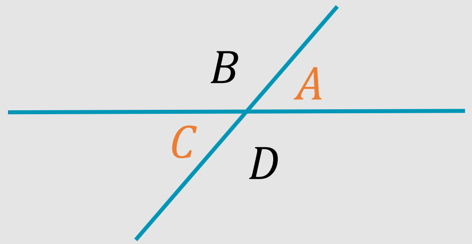
Angles at a point on a straight line

Vertically opposite angles

For intersecting lines, angles which are directly opposite each other are called vertically opposite angles.

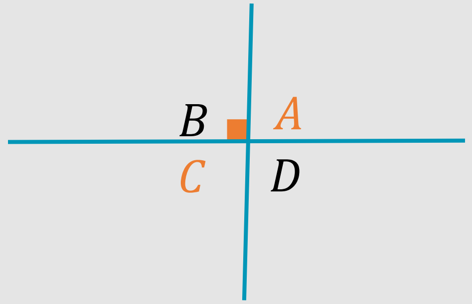
Vertically opposite angles are equal in size.

$$A = C$$
$$B = D$$



Note:

If 2 lines are perpendicular to each other, all 4 angles are equal.



$$A = B = C = D = 90°$$

Angle properties

Angles at a point

Angles at a point on a straight line

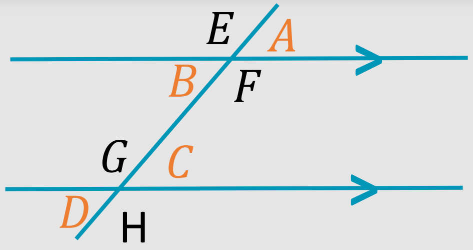
Vertically opposite angles

Angles formed within parallel lines

When two parallel lines are cut by a third line:
8 angles are obtained. Acute angles are equal and obtuse angles are equal.

$$A = B = C = D$$

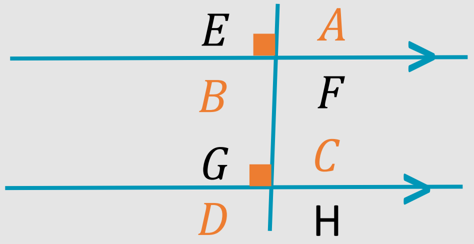
$$E = F = G = H$$



Note:

If a line is perpendicular to 2 parallel lines, all 8 angles are equal.

$$A = B = C = D = E = F = G = H = 90°$$



Angle properties

Angles at a point

Angles at a point on a straight line

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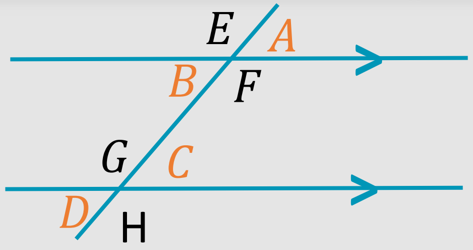


Corresponding angles

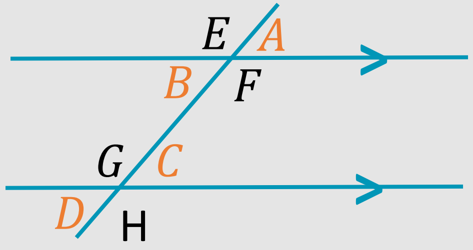
When two parallel lines are cut by a third line, then angles in the corresponding positions are equal in size.

$F = H$

$E = G$

$A = C$

$B = D$



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Angles at a point

Angles at a point on a straight line

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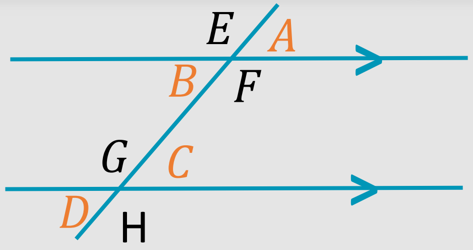


Alternate angles

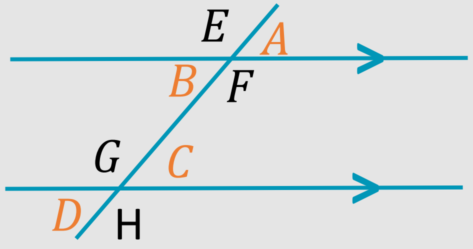
When two parallel lines are cut by a third line, then angles in alternate positions are equal in size.

$B = C$

$A = D$

$E = H$

$F = G$



Angle properties

Angles at a point

Angles at a point on a straight line

Vertically opposite angles

Angles formed within parallel lines

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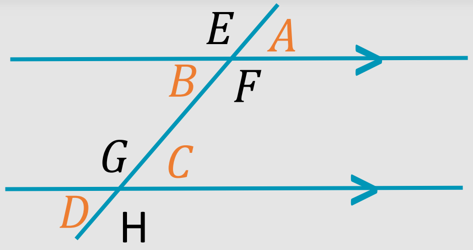
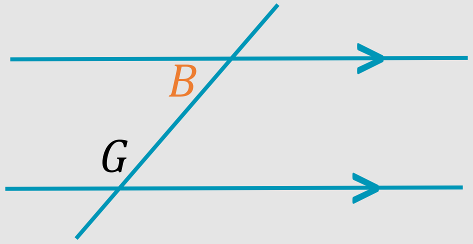


Co-interior angles (also called allied angles)

When two parallel lines are cut by a third line, then co-interior angles are supplementary

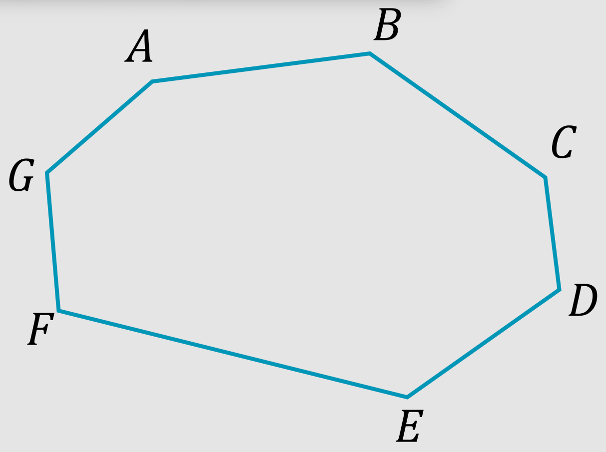
$B + G = 180$



The interior angles of a polygon

The sum of the interior angles of any n-sided polygon is $(\boldsymbol{n} - \mathbf{2}) \times \mathbf{180^o}$.

$$A + B + C + D + E + F + G = (7 - 2) \times 180$$
$$= 900$$



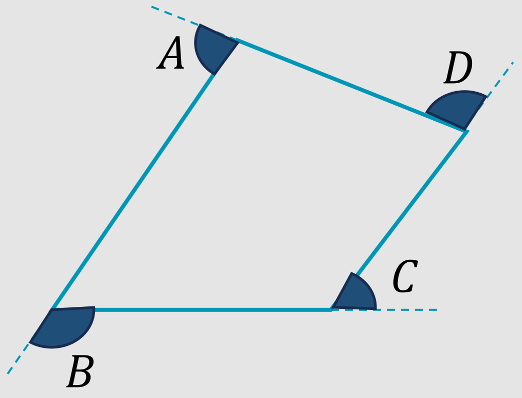
Angle properties of irregular polygons and regular polygons

The interior angles of a polygon

The exterior angles of a polygon

The sum of the exterior angles of any polygon is always $\mathbf{360^o}$.

$$A + B + C + D = 360$$



Angle properties of irregular polygons and regular polygons

The interior angles of a polygon

The exterior angles of a polygon

The interior angles of a regular polygon

The size of the interior angles in any n-sided regular polygon is $\dfrac{(n-2)\times 180^o}{n}$

Angle properties of irregular polygons and regular polygons

The interior angles of a polygon

The exterior angles of a polygon

The interior angles of a regular polygon

The exterior angles of a regular polygon

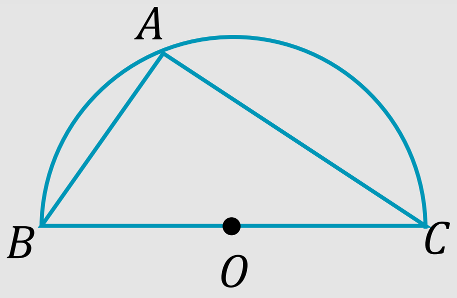
The size of the exterior angles in any n-sided regular polygon is $\frac{360^o}{n}$

Circle theorems

Angle in a semi-circle

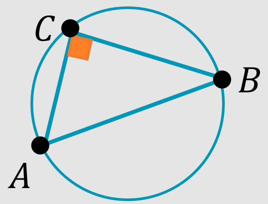
The angle in a semi-circle is a right angle.

$ABC = 90°$



Note:

If line segment AB subtends a right angle at C, then the circle through A, B, and C has diameter AB.

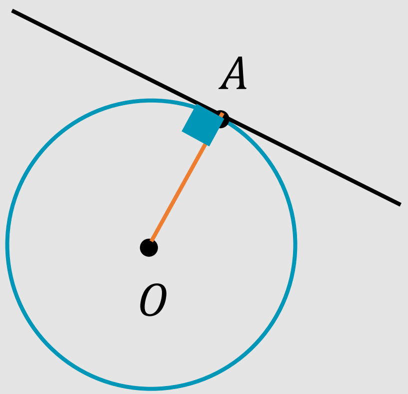


Circle theorems

Angle in a semi-circle

Angle between tangent and radius of a circle

A tangent is a line that touches the circle at a single point on its circumference. If we draw a radius that meets the circumference at the same point, the angle between the radius and the tangent will always be exactly 90°.



Circle theorems

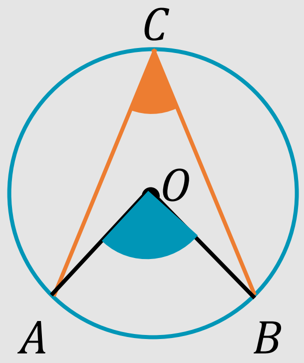
Angle in a semi-circle

Angle between tangent and radius of a circle

Angle at the centre of a circle

The angle at the centre of a circle is twice the angle on the circle subtended by the same arc.

$$AOB = 2ACB$$



Circle theorems

Angle in a semi-circle

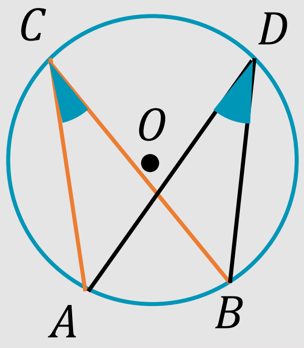
Angle between tangent and radius of a circle

Angle at the centre of a circle

Angles in the same segment

Angles in the same segment of a circle are equal.

$$ACB = ADB$$



Circle theorems

Angle in a semi-circle

Angle between tangent and radius of a circle

Angle at the centre of a circle

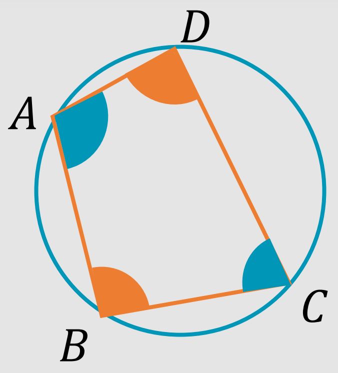
Angles in the same segment

Opposite angles of a cyclic quadrilateral

The opposite angles of a cyclic quadrilateral are supplementary.

$$BAD + BCD = 180°$$

$$ADC + ABC = 180°$$



Circle theorems

Angle in a semi-circle

Angle between tangent and radius of a circle

Angle at the centre of a circle

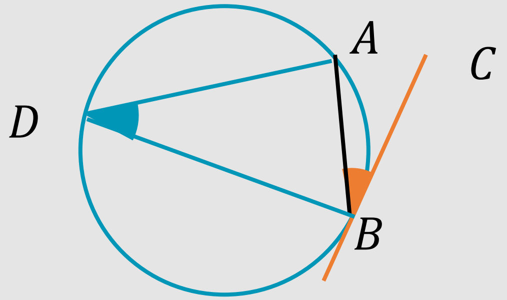
Angles in the same segment

Opposite angles of a cyclic quadrilateral

Alternate segment

Alternate segment theorem:

The angle formed between the tangent and the chord through the point of contact of the tangent is equal to the angle formed by the chord in the alternate segment.



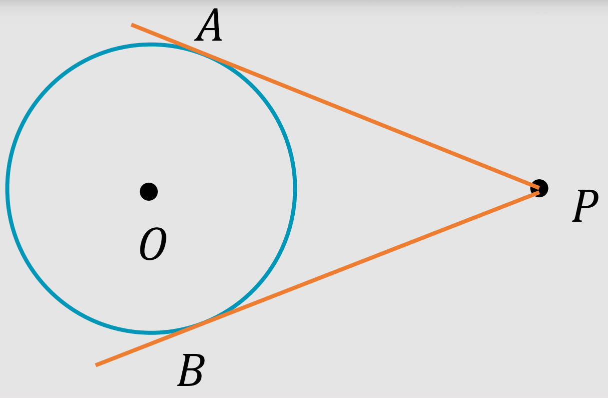
$$ADB = ABC$$

Use the following symmetry properties of circles

Tangents from an external point

Tangents from an external point are equal in length.

$$AP = BP$$

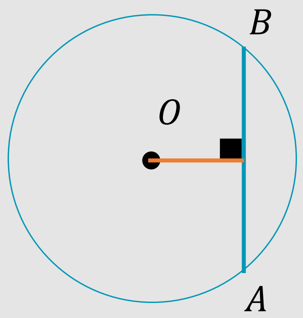


Use the following symmetry properties of circles

Tangents from an external point

Perpendicular bisector of a chord

The perpendicular bisector of a chord of a circle passes through its centre.



Use the following symmetry properties of circles

Tangents from an external point

Perpendicular bisector of a chord

Equal chords are equidistant from the centre

Equal chords are equidistant from the centre.

$$AB = CD \quad \Leftrightarrow \quad OM = ON$$