Differentiation

Past papers (2024) - With Answer

(c) (i) Find the coordinates of the turning points of the graph of $y = 10 + 9x^2 - 2x^3$.
You must show all your working.

$$\frac{dy}{dx} = 18x - 6x^2$$

$$18x - 6x^2 = 0$$

$$6x(3 - x) = 0 \quad \begin{cases} 6x = 0 \rightarrow x = 0 \\ 3 - x = 0 \rightarrow x = 3 \end{cases} \qquad f''(x) = 18 - 12x$$

$$x = 0 \qquad y = 10 + 9(0)^2 - 2(0)^3 \rightarrow y = 10$$

$$x = 3 \qquad y = 10 + 9(3)^2 - 2(3)^3 \rightarrow y = 37$$

(0 , 10) and (3 , 37) [5]

(ii) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

$$f''(x) = 18 - 12x$$

$x = 0$ $\quad f''(0) = 18 - 12(0) = 18 > 0 \quad Minimum.$

$x = 3$ $\quad f''(3) = 18 - 12(3) = -18 < 0 \quad Maximum.$

[3]

8(c)(i)	$18x - 6x^2$ isw	**B2**	**B1** for one correct term $18x$ or $-6x^2$ seen
	setting *their* derivative = 0 or $\frac{dy}{dx} = 0$	**M1**	**Dep** on at least B1 earned or *their* derivative = $\pm 18x \pm 6x^2$
	(0, 10) and (3, 37)	**B2**	**B1** for $x = 0$ and $x = 3$ or for (0, 10) or (3, 37)
8(c)(ii)	(0, 10) minimum with correct reason AND (3, 37) maximum with correct reason	**3**	Reasons could be e.g. 1 A reasonable sketch of a negative cubic 2 Correct use of 2nd derivative = $-12(0) + 18 = 18$, $18 > 0$, so (0, 10) is a minimum oe. 2nd derivative = $-12(3) + 18 = -18$, $-18 < 0$ so (3, 37) is a maximum oe. 3 Evaluates correctly values of y on both sides of both correct stationary points 4 Finds gradient on each side of both correct stationary points. **B2** for 1 correct with correct reason for that stationary point or for **both** x-values correct and reasonable sketch of a negative cubic, or for correct substitution and evaluation of **both** of *their* x-values into *their* second derivative

(b) (i) Find the derivative of $y = x^2 - 3x - 28$.

$$\frac{dy}{dx} = 2x - 3$$

..... [2]

(ii) Find the coordinates of the turning point of $y = x^2 - 3x - 28$.

$2x - 3 = 0 \quad \rightarrow \quad 2x - 3 + 3 = 0 + 3$

$\rightarrow \quad 2x = 3 \qquad \frac{2x}{2} = \frac{3}{2} \qquad x = \frac{3}{2} = 1.5$

$y = (1.5)^2 - 3(1.5) - 28 = -30.25$

(1.5 , −30.25) [3]

8(b)(i)	$2x - 3$	**2**	**B1** for $2x + k$ or $kx^{[p]} - 3$
8(b)(ii)	(1.5, –30.25) oe	**3**	**B2** for $x = 1.5$ or **M1** for *their* **(b)(i)** $= 0$ or for $(x - 1.5)^2$

11 **(a)** The point $(-1, 6)$ lies on a curve.

This curve has the derived function $\frac{dy}{dx} = -4x^3 - 9x^2 + 5$.

Show that $(-1, 6)$ is a stationary point of the curve.

$$f'(-1) = -4(-1)^3 - 9(-1)^2 + 5 = 0$$

[2]

(b) A different curve has equation $y = 2x^3 - 6x + 8$.

(i) Calculate the gradient of the tangent to this curve at the point $(-2, 2)$.

$Gradient = f'(x_A)$

$f'(x) = 6x^2 - 6$

$f'(-2) = 6(-2)^2 - 6 = 18$

..... [3]

(ii) Find the x-coordinates of the stationary points of this curve.

$f'(x) = 6x^2 - 6$

$6x^2 - 6 = 0 \qquad 6x^2 = 6 \qquad x^2 = 1 \qquad x = \pm\sqrt{1} \qquad x = \pm 1$

$x =$ ..... and $x =$ ..... [2]

11(a)	$-4(-1)^3 - 9(-1)^2 + 5$ or better	**M1**	
	$= 0$ [so stationary point]	**A1**	with no errors
11(b)(i)	18	**3**	**B2** for $6x^2 - 6$ isw OR **B1** for $6x^2 + k$ (any k) isw or $px^2 - 6$ isw ($p \neq 0$) or $6x^2 - 6 + 8$ **M1**dep on B1 for $x = -2$ substituted into *their* $\frac{dy}{dx}$
11(b)(ii)	1 and –1	**2**	**M1** for $6x^2 - 6 = 0$ oe seen or for *their* $\frac{dy}{dx} = 0$ if B1 scored in part (b)(i)

10 $y = x^7 - 7x^6$

(a) Find the derivative of y with respect to x.

$$\frac{dy}{dx} = 7x^6 - 42x^5$$

..... [2]

(b) Find the equation of the tangent to the graph of $y = x^7 - 7x^6$ at the point where $x = -1$.
Give your answer in the form $y = mx + c$.

$y - y_A = m(x - x_A)$ *Gradient* $= f'(x_A)$ $m = f'(-1) = 7(-1)^6 - 42(-1)^5 = 49$

$y - (-8) = 49(x - (-1))$ $y_A = (-1)^7 - 7(-1)^6 = -8$

$y = 49x - 41$

$y =$ ..... [4]

(c) The graph of $y = x^7 - 7x^6$ has two turning points.

Find the coordinates of these points.
You must show all your working.

$f'(x) = 7x^6 - 7 \times 6x^5$

$7x^6 - 7 \times 6x^5 = 0$

$7x^5(x - 6) = 0$

- $7x^5 = 0 \quad \rightarrow \quad x = 0$
- $(x - 6) = 0 \quad \rightarrow \quad x = 6$

$x = 0 \qquad y = (0)^7 - 7(0)^6 = 0$

$x = 6 \qquad y = (6)^7 - 7(6)^6 = -46656$

(0 , 0)

(6 , −46656) [5]

10(a)	$7x^6 - 42x^5$ final answer	**2**	**B1** for one correct term $7x^6$ or $42x^5$ or for $7x^6 - 42x^5$ seen and spoiled
10(b)	$49x + 41$	**4**	**M1** for substituting $x = -1$ into $[y =] x^7 - 7x^6$ **M1** for $x = -1$ substituted in *their* (a) or the correct derivative to give *their m* **M1** for *their* $-8 = (\textit{their } m)(-1) + c$ oe
10(c)	(0, 0) (6, –46 656)	**5**	**B4** for (6, –46 656) or **B3** for $x = 0$ and 6 **OR** **M1** for *their* $\frac{dy}{dx} = 0$ or stating $\frac{dy}{dx} = 0$ and **M1** for a correct method to solve *their* $7x^6 - 42x^5$