

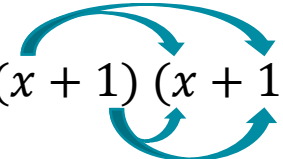
Question

$$f(x) = x + 1 \quad g(x) = x^3 + 3 \quad h(x) = x^2 + 2$$

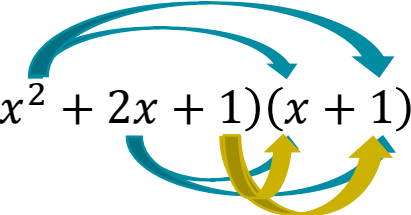
Find $gf(x) + h(x)$. Give your answer in its simplest form.

$$gf(x) = g(f(x))$$

$$= (x + 1)^3 + 3$$

$$= (x + 1)(x + 1)(x + 1) + 3$$


$$= (x^2 + x + x + 1)(x + 1) + 3$$

$$= (x^2 + 2x + 1)(x + 1) + 3$$


$$= x^3 + x^2 + 2x^2 + 2x + x + 1 + 3$$

$$= x^3 + 3x^2 + 3x + 4$$

$$gf(x) + h(x) =$$

$$x^3 + 3x^2 + 3x + 4 + x^2 + 2$$

$$gf(x) + h(x) = x^3 + 4x^2 + 3x + 6$$

Question

$$f(x) = x + 3 \quad g(x) = x^2 - 4$$

$$\text{Solve } fg(x) = gf(2x - 1).$$

$$fg(x) = gf(2x - 1)$$

$$x^2 - 1 = 4x^2 + 4x$$

$$0 = 4x^2 + 4x - x^2 + 1$$

$$3x^2 + 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-1}{3}$$
$$x = -1$$

$$fg(x) = f(g(x))$$

$$fg(x) = f(x^2 - 4)$$

$$= (x^2 - 4) + 3$$

$$= x^2 - 1$$

$$gf(2x - 1) = g(f(2x - 1))$$

$$f(2x - 1) = (2x - 1) + 3$$

$$f(2x - 1) = 2x + 2$$

$$gf(2x - 1) = g(2x + 2)$$

$$= (2x + 2)^2 - 4$$

$$= (2x + 2)(2x + 2) - 4$$

$$= (4x^2 + 4x + 4x + 4) - 4$$

$$= 4x^2 + 8x$$

Find the inverse of the function $g(x) = 3x - 2$, then calculate $f^{-1}(-1)$.

$$y = 3x - 2$$

$$x = 3y - 2$$

$$x + 2 = 3y$$

$$y = \frac{(x + 2)}{3}$$

$$f^{-1}(x) = \frac{(x + 2)}{3}$$

$$f^{-1}(-1) = \frac{(-1 + 2)}{3}$$

$$f^{-1}(-1) = \frac{1}{3}$$

find the inverse of the function $h(x) = \frac{2x - 3}{4x + 1}$.

$$h(x) = \frac{2x - 3}{4x + 1}$$

$$y = \frac{2x - 3}{4x + 1}$$

$$x = \frac{2y - 3}{4y + 1}$$

$$x \times (4y + 1) = \frac{2y - 3}{\cancel{4y + 1}} \times \cancel{(4y + 1)}$$

$$4xy + x = 2y - 3$$

$$4xy - 2y = -3 - x$$

$$y(4x - 2) = -3 - x$$

$$\frac{y(4x - 2)}{(4x - 2)} = \frac{-3 - x}{(4x - 2)}$$

$$y = \frac{-3 - x}{(4x - 2)}$$

$$f^{-1}(x) = \frac{-3 - x}{(4x - 2)}$$

$$f(x) = 1 - 2x^3$$

Find $f^{-1}(x)$.

$$f(x) = 1 - 2x^3$$

$$y = 1 - 2x^3$$

$$x = 1 - 2y^3$$

$$x - 1 = -2y^3$$

$$\frac{x - 1}{-2} = y^3$$

$$\sqrt[3]{\frac{x - 1}{-2}} = y$$

$$\sqrt[3]{\frac{x - 1}{-2}} = f^{-1}(x)$$

Find the inverse of the function $g(x) = \sqrt[3]{x - 3}$.

$$y = \sqrt[3]{x - 3}$$

$$x = \sqrt[3]{y - 3}$$

$$x^3 = y - 3$$

$$x^3 + 3 = y$$

$$x^3 + 3 = g^{-1}(x)$$

$$f(x) = 3x - 2.$$

Find $f^{-1}f(x - 1)$.

$$f(x) = 3x - 2$$

$$y = 3x - 2$$

$$x = 3y - 2$$

$$x + 2 = 3y$$

$$\frac{x + 2}{3} = y$$

$$\frac{x + 2}{3} = f^{-1}(x)$$

$$f(x) = 3x - 2 \longrightarrow f(x - 1) = 3(x - 1) - 2 \\ = 3x - 5$$

$$f^{-1}f(x - 1) = f^{-1}(f(x - 1))$$

$$= f^{-1}(3x - 5)$$

$$f^{-1}(f(x - 1)) = \frac{(3x - 5) + 2}{3} \\ = \frac{3x - 3}{3} = \frac{3x}{3} - \frac{3}{3} \\ = x - 1$$

$$f^{-1}f(x + 3) = x - 1$$

$$f(x) = \frac{3x - 1}{2}.$$

$$\text{Solve } f^{-1}(x) + 2x = 0$$

$$f(x) = \frac{3x - 1}{2}$$

$$y = \frac{3x - 1}{2}$$

$$x = \frac{3y - 1}{2}$$

$$2x = 3y - 1$$

$$2x + 1 = 3y$$

$$\frac{2x + 1}{3} = y$$

$$\frac{2x + 1}{3} = f^{-1}(x)$$

$$f^{-1}(x) + 2x = 0$$

$$\frac{2x + 1}{3} + 2x = 0$$

$$\frac{2x + 1}{3} + 2x \times \frac{3}{3} = 0$$

$$\frac{2x + 1 + 6x}{3} = 0$$

$$2x + 1 + 6x = 0$$

$$8x + 1 = 0$$

$$x = \frac{-1}{8}$$

$$f(x) = 5^x.$$

Find x when $f^{-1}(x) = 3$

Note

$y = f^{-1}(x)$ which is equal to $x = f(y)$.

$$x = f(y)$$

$$x = 5^3$$

$$x = 125$$

$$f(x) = 7^{x+1}.$$

Find x when $f^{-1}(x) = 2$

Note

$y = f^{-1}(x)$ which is equal to $x = f(y)$.

$$x = f(y)$$

$$x = 7^{x+1}$$

$$x = 7^{(2)+1}$$

$$x = 343$$

$$f(x) = \frac{1-x}{2+x}, \text{ and } g^{-1}(x) = f(x).$$

Find $g(x)$.

Note

$$(f^{-1})^{-1} = f$$

$$g^{-1}(x) = f(x)$$

$$f(x) = \frac{1-x}{2+x}$$

$$x(2+y) = \frac{1-y}{2+y} \times (2+y)$$

$$(g^{-1}(x))^{-1} = (f(x))^{-1}$$

$$y = \frac{1-x}{2+x}$$

$$2x + xy = 1 - y$$

$$xy + y = 1 - 2x$$

$$g(x) = f^{-1}(x)$$

$$x = \frac{1-y}{2+y}$$

$$y(x+1) = 1 - 2x$$

$$y = \frac{1-2x}{x+1}$$

$$f^{-1}(x) = \frac{1-2x}{x+1}$$

$$g(x) = f^{-1}(x) = \frac{1-2x}{x+1}$$

$$f(x) = 5^{2x-1}.$$

Find $ff^{-1}(x)$

Note

$$ff^{-1} = f^{-1}f = x$$

$$ff^{-1} = x$$