Question

$f(x) = x + 1$ $g(x) = x^3 + 3$ $h(x) = x^2 + 2$

Find $gf(x) + h(x)$. Give your answer in its simplest form.

$gf(x) = g(f(x))$

$= (x+1)^3 + 3$

$= (x+1)(x+1)(x+1) + 3$

$= (x^2 + x + x + 1)(x+1) + 3$

$= (x^2 + 2x + 1)(x+1) + 3$

$= x^3 + x^2 + 2x^2 + 2x + x + 1 + 3$

$= x^3 + 3x^2 + 3x + 4$

$gf(x) + h(x) =$

$x^3 + 3x^2 + 3x + 4 + x^2 + 2$

$gf(x) + h(x) = x^3 + 4x^2 + 3x + 6$

Question

$f(x) = x + 3 \qquad g(x) = x^2 - 4$

Solve $fg(x) = gf(2x-1)$.

$fg(x) = gf(2x-1)$

$x^2 - 1 = 4x^2 + 4x$

$0 = 4x^2 + 4x - x^2 + 1$

$3x^2 + 4x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \frac{-1}{3}$

$x = -1$

$fg(x) = f(g(x))$

$fg(x) = f(x^2 - 4)$

$= (x^2 - 4) + 3$

$= x^2 - 1$

$gf(2x-1) = g(f(2x-1))$

$f(2x-1) = (2x-1) + 3$

$f(2x-1) = 2x + 2$

$gf(2x-1) = g(2x+2)$

$= (2x+2)^2 - 4$

$= (2x+2)(2x+2) - 4$

$= (4x^2 + 4x + 4x + 4) - 4$

$= 4x^2 + 8x$

Question 3

Find the inverse of the function g(x) = 3x – 2, then calculate $f^{-1}(-1)$.

$$y = 3x - 2$$
$$x = 3y - 2$$
$$x + 2 = 3y$$
$$y = \frac{(x + 2)}{3}$$
$$f^{-1}(x) = \frac{(x + 2)}{3}$$

$$f^{-1}(-1) = \frac{(-1 + 2)}{3}$$
$$f^{-1}(-1) = \frac{1}{3}$$

Question 4

find the inverse of the function $h(x) = \frac{2x-3}{4x+1}$.

$$h(x) = \frac{2x - 3}{4x + 1}$$

$$y = \frac{2x - 3}{4x + 1}$$

$$x = \frac{2y - 3}{4y + 1}$$

$$x \times (4y + 1) = \frac{2y - 3}{4y + 1} \times (4y + 1)$$

$$4xy + x = 2y - 3$$

$$4xy - 2y = -3 - x$$

$$y(4x - 2) = -3 - x$$

$$\frac{y(4x-2)}{(4x-2)} = \frac{-3-x}{(4x-2)}$$

$$y = \frac{-3-x}{(4x-2)}$$

$$f^{-1}(x) = \frac{-3-x}{(4x-2)}$$

Question 5

$\boldsymbol{f(x) = 1 - 2x^3}$

Find $\boldsymbol{f^{-1}(x)}$.

$$f(x) = 1 - 2x^3$$

$$y = 1 - 2x^3$$

$$x = 1 - 2y^3$$

$$x - 1 = -2y^3$$

$$\frac{x-1}{-2} = y^3$$

$$\sqrt[3]{\frac{x-1}{-2}} = y \qquad \sqrt[3]{\frac{x-1}{-2}} = f^{-1}(x)$$

Question 6

Find the inverse of the function g(x) = $\sqrt[3]{x-3}$**.**

$$y = \sqrt[3]{x-3}$$

$$x = \sqrt[3]{y-3}$$

$$x^3 = y - 3$$

$$x^3 + 3 = y$$

$$x^3 + 3 = g^{-1}(x)$$

Question 7

$\boldsymbol{f(x) = 3x - 2.}$

Find $\boldsymbol{f^{-1}f(x-1)}$**.**

$f(x) = 3x - 2$

$y = 3x - 2$

$x = 3y - 2$

$x + 2 = 3y$

$\dfrac{x+2}{3} = y$

$\dfrac{x+2}{3} = f^{-1}(x)$

$f(x) = 3x - 2 \longrightarrow f(x-1) = 3(x-1) - 2$

$= 3x - 5$

$f^{-1}f(x-1) = f^{-1}(f(x-1))$

$= f^{-1}(3x-5)$

$f^{-1}\big(f(x-1)\big) = \dfrac{(3x-5)+2}{3}$

$= \dfrac{3x-3}{3} = \dfrac{3x}{3} - \dfrac{3}{3}$

$= x - 1$

$f^{-1}f(x+3) = x - 1$

Question 8

$$\boldsymbol{f(x) = \frac{3x-1}{2}}\ .$$

Solve $\boldsymbol{f^{-1}(x) + 2x = 0}$

$$f(x) = \frac{3x-1}{2}$$

$$y = \frac{3x-1}{2}$$

$$x = \frac{3y-1}{2}$$

$$2x = 3y - 1$$

$$2x + 1 = 3y$$

$$\frac{2x+1}{3} = y \qquad \frac{2x+1}{3} = f^{-1}(x)$$

$$f^{-1}(x) + 2x = 0$$

$$\frac{2x+1}{3} + 2x = 0$$

$$\frac{2x+1}{3} + 2x \times \frac{3}{3} = 0$$

$$\frac{2x+1+6x}{3} = 0$$

$$2x + 1 + 6x = 0$$

$$8x + 1 = 0 \qquad x = \frac{-1}{8}$$

Question 9

$\boldsymbol{f(x) = 5^x}$.

Find x when $\boldsymbol{f^{-1}(x) = 3}$

Note

$y = f^{-1}(x)$ which is equal to $x = f(y)$.

$x = f(y)$

$x = 5^3$

$x = 125$

Question 10

$\boldsymbol{f(x) = 7^{x+1}}$.

Find x when $\boldsymbol{f^{-1}(x) = 2}$

Note

$y = f^{-1}(x)$ which is equal to $x = f(y)$.

$$x = f(y)$$

$$x = 7^{x+1}$$

$$x = 7^{(2)+1}$$

$$x = 343$$

Question 11

$\boldsymbol{f(x)=\frac{1-x}{2+x}}$ **, and** $\boldsymbol{g^{-1}(x)=f(x)}$.

Find $\boldsymbol{g(x)}$.

Note $(f^{-1})^{-1}=f$

$$g^{-1}(x)=f(x)$$

$$(g^{-1}(x))^{-1}=(f(x))^{-1}$$

$$g(x)=f^{-1}(x)$$

$$f(x)=\frac{1-x}{2+x}$$

$$y=\frac{1-x}{2+x}$$

$$x=\frac{1-y}{2+y}$$

$$x(2+y)=\frac{1-y}{2+y}\times(2+y)$$

$$2x+xy=1-y$$

$$xy+y=1-2x$$

$$y(x+1)=1-2x$$

$$y=\frac{1-2x}{x+1}$$

$$f^{-1}(x)=\frac{1-2x}{x+1}$$

$$g(x)=f^{-1}(x)=\frac{1-2x}{x+1}$$

Question 12

$f(x) = \mathbf{5}^{\mathbf{2}x-1}$.

Find $\boldsymbol{ff^{-1}(x)}$

Note $ff^{-1} = f^{-1}f = x$

$$ff^{-1} = x$$