Sketch functions

In IGCSE, sketching graphs involves drawing a freehand diagram of a function that shows the key points and the general shape of the graph. The diagram does not need to be perfectly scaled, but it should have the right shape and the values of any intercepts should be clearly labeled.

On the other hand, plotting graphs involves producing a graph by marking points accurately on a grid from provided data and then drawing a line of best fit through these points 34. The difference between these two command words is the use of scales. A plotted graph has scaled axes, while a sketch doesn't have to be, but both times the axes should be clearly labeled.

constante fonction

$\boldsymbol{y = a}$ **or** $\boldsymbol{f(x) = a}$

A constant function is a function that always has the same output, irrespective of the input value. It is the simplest type of real-valued function and can be represented by a horizontal line in the plane.

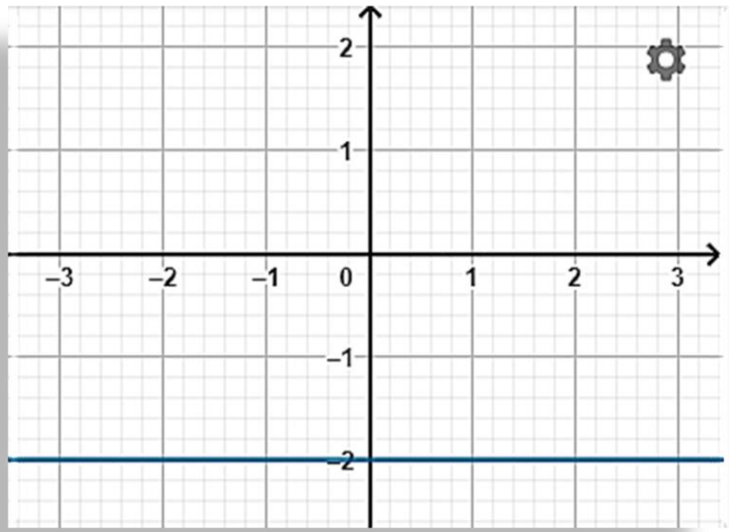
The graph of a constant function is a straight line parallel to the x-axis.

To sketch a constant function, we need to know the value of the constant.
Once we know the value of the constant, we can draw a horizontal line at that value on the y-axis.

For example:

Sketch the graph of the $f(x) = -2$.

Here the constant is -2,
we can draw a horizontal line at $y = -2$.



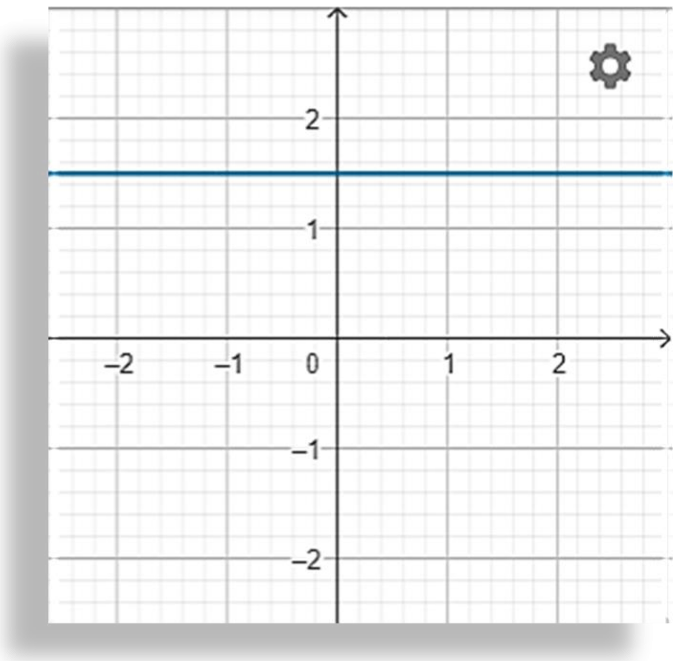
Example

Sketch the graph of the function $2y - 3 = 0$.

$$2y - 3 = 0$$

$$2y = 3$$

$$y = \frac{3}{2} = 1.5$$



Linear functions

$$y = ax + b \qquad \textbf{or} \qquad f(x) = ax + b$$

This function is called **the linear function.**

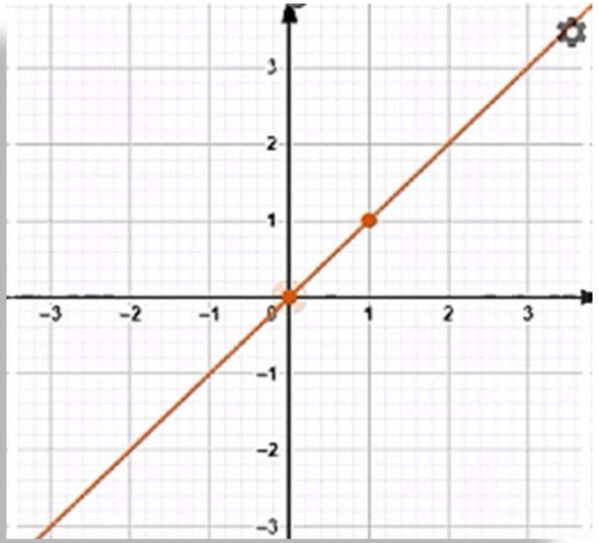
The graph of a linear function is **a straight line.**

To sketch the linear function $y = ax + b$, we create a table with two key points, the x-coordinate of these two points will be 0 and 1.

For example: Sketch the graph of the $y = x$.

x	$y = x$
0	$y = 0$
1	$y = 1$

The basic shape of the linear functions is a straight line, and points (0,0) and (1,1) are key points in their graphs.

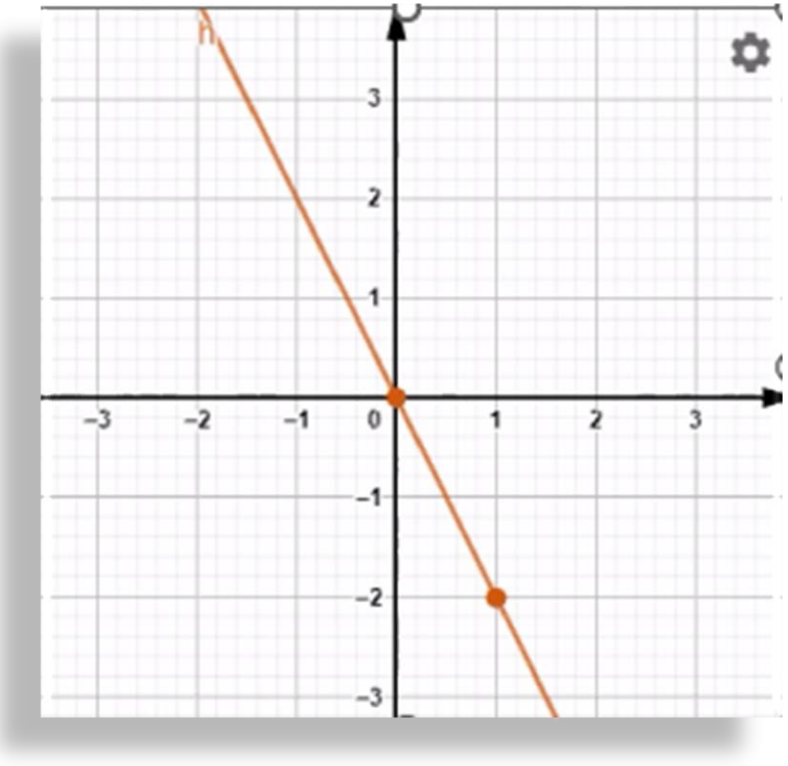


Example Sketch the graph of the function $\boldsymbol{y + 2x = 0}$.

This function is called **the linear function.**

The graph of a linear function is **a straight line.**

x	$y = -2x$
0	$y = -2(0) = 0$
1	$y = -2(1) = -2$



Sketch
Quadratic functions

$$\boldsymbol{y = ax^2 + bx + c} \qquad \textbf{or} \qquad \boldsymbol{f(x) = ax^2 + bx + c}$$

This function is called **the quadratic function.**

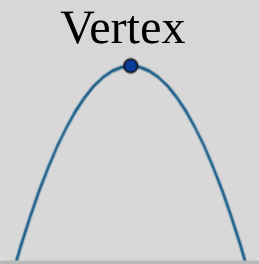
The graph of a quadratic function **is a parabola.**

The quadratic function curve has a turning point (vertex)

In a quadratic function of the form $y = ax^2 + bx + c$, the **maximum** or **minimum** value of the function occurs at the **vertex** (**turning point**) of the parabola.

- If $a > 0$, then the parabola opens upwards, the vertex is the **minimum**, and the graph is concave upwards.

- If $a < 0$, then the parabola opens downwards, the vertex is the **maximum**, and the graph is concave downwards.



The maximum or minimum value of the quadratic function

The general steps to find the maximum or the minimum value of the quadratic function in the form $y = ax^2 + bx + c$:

1. If necessary, rearrange the quadratic into the form $y = ax^2 + bx + c$
2. The coordinates of the vertex point (turning point) are $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$
3.
   - If $a > 0$, then the parabola opens upwards, the vertex is the minimum, and the graph is concave upwards.
   - If $a < 0$, then the parabola opens downwards, the vertex is the maximum, and the graph is concave downwards.

The general steps to find the maximum or the minimum value of the quadratic function in the form $y = a(x + p)^2 + q$:

1. The coordinates of the vertex point (turning point) are $(-p, q)$
2.
   - If $a > 0$, then the parabola opens upwards, the vertex is the minimum, and the graph is concave upwards.
   - If $a < 0$, then the parabola opens downwards, the vertex is the maximum, and the graph is concave downwards.

Example

Find the maximum or minimum point in the following function.

$$y = 2x^2 - x - \frac{1}{5}$$

1 **If necessary, rearrange the quadratic into the form y $= ax^2 + bx + c$**

$$y = 2x^2 - x - \frac{1}{5}$$

2 **The coordinates of the vertex point (turning point) are $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$**

x-coordinate $= \frac{-b}{2a}$

$$= \frac{-(-1)}{2(2)} = \frac{1}{4} = 0.25$$

y-coordinate $= f\left(\frac{-b}{2a}\right)$

$$f\left(\frac{1}{4}\right) = 2\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) - \frac{1}{5} = -0.325$$

vertex $(0.25, -0.325)$

3
- **If $a > 0$, then the parabola opens upwards, the vertex is the minimum, and the graph is concave upwards.**
- **If $a < 0$, then the parabola opens downwards, the vertex is the maximum, and the graph is concave downwards.**

$a = 2 \quad > 0$ then the parabola opens upwards, the vertex $(0.25, -0.325)$ is the minimum point.

The **minimum value** of the function is -0.325

Example

Find the maximum or minimum point in the following function.

$$y = 1 - 2x^2$$

$y = 1 - 2x^2$ $\quad\quad$ $y = -2x^2 + 1$

x-coordinate = $\frac{-b}{2a}$ $\quad\quad$ y-coordinate $= f(\frac{-b}{2a})$

$= \frac{-(0)}{2(-2)} = 0$ $\quad\quad$ $f(0) = -2(0)^2 + 1 = 1$ $\quad\quad$ vertex (0,1)

$a = -2 < 0$ then the parabola opens downwards, the vertex (0,1) is the maximum point.

The **maximum value** of the function is 1

Example

Find the maximum or minimum point in the following function.

$$y = (x+1)^2$$

$y = (x+1)^2$

$y = (x+1)^2 + 0$

$\mathbf{y = a(x+p)^2 + q}$

The coordinates of the vertex point (turning point) are $(-p, q)$

x-coordinate $= -(\mathbf{1})$ y-coordinate $= 0$ vertex $(-1,0)$

$a = 1 > 0$ then the parabola opens upwards, the vertex $(-1,0)$ is the minimum point.

The **minimum value** of the function is 0

Sketch Quadratic functions

The general steps to sketch a quadratic function:

1. Find the coordinates of the turning point (maximum/minimum).
2. Find the intersection of the graph with the coordinate axes.
   - To find the x-axis intercept of a function, substitute $y = 0$ and solve for x.
   - To find the y-axis intercept of a function, substitute $x = 0$ and solve for y.
3. Enter these points in the coordinate plane and connect them parabolically.

Example Sketch the graph of $y = x^2 - 2x - 1$.

Step 1

The coordinate of the turning point.

$$\begin{cases} x = 1 \\ y = -2 \end{cases}$$

Step 2

Find the intersection of the graph with the coordinate axes.

- To find the x-axis intercept of a function, substitute y = 0 and solve for x.
- To find the y-axis intercept of a function, substitute x = 0 and solve for y.

$$\boldsymbol{y = 0}$$

$$0 = x^2 - 2x - 1 \qquad \boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = -0.41$$

$$x = 2.41$$

$$\boldsymbol{x = 0}$$

$$y = x^2 - 2x - 1$$

$$y = (0)^2 - 2(0) - 1$$

$$y = -1$$

The coordinates of the points where the graph intersects the x-axis.

$$\begin{cases} x = -0.41 \\ y = 0 \end{cases} \quad \begin{cases} x = 2.41 \\ y = 0 \end{cases}$$

The coordinates of the point where the graph intersects the y-axis.

$$\begin{cases} x = 0 \\ y = -1 \end{cases}$$

Example

Sketch the graph of $y = x^2 - 2x - 1$.

Step 1

The coordinate of the turning point.

$$\begin{cases} x = 1 \\ y = -2 \end{cases}$$

Step 2

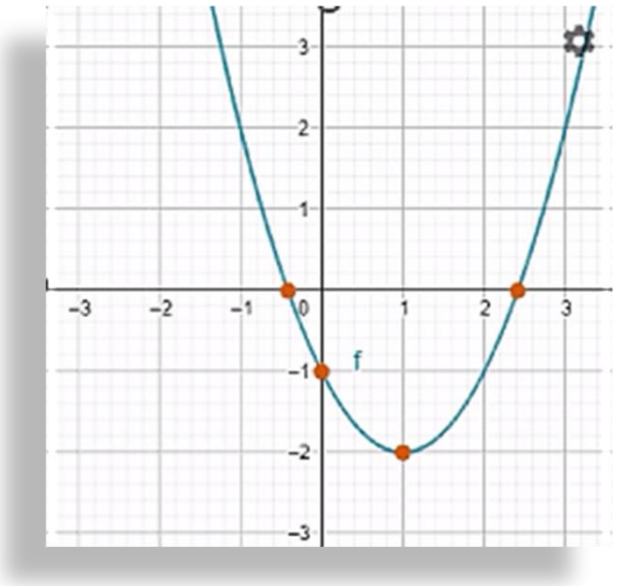
The coordinates of the points where the graph intersects the x-axis.

$$\begin{cases} x = -0.41 \\ y = 0 \end{cases} \quad \begin{cases} x = 2.41 \\ y = 0 \end{cases}$$

The coordinates of the point where the graph intersects the y-axis.

$$\begin{cases} x = 0 \\ y = -1 \end{cases}$$

Step 3



Example Sketch the graph of $y = 2 - 3(x+1)^2$.

$y = 2 - 3(x+1)^2$ $\quad\quad$ $y = -3(x+1)^2 + 2$

$$\mathbf{y = a(x+p)^2 + q}$$

The coordinates of the vertex point (turning point) are $(-p, q)$

x-coordinate = $-(\mathbf{1})$ $\quad\quad$ y-coordinate =2 $\quad\quad$ vertex $(-1,2)$

$a = -3 < 0$ then the parabola opens downwards, the vertex $(-1,0)$ is the maximum point.

Example

Sketch the graph of $y = 2 - 3(x+1)^2$.

The coordinates of the turning point.

$$\begin{cases} x = -1 \\ y = 2 \end{cases}$$

$$\boldsymbol{y = 0}$$

$$0 = 2 - 3(x+1)^2$$

$$\frac{2}{3} = (x+1)^2$$

$$\pm\sqrt{\frac{2}{3}} = x + 1 \qquad -1 \pm \sqrt{\frac{2}{3}} = x$$

$$x = -1.82$$

$$x = -0.18$$

The coordinates of the points where the graph intersects the x-axis.

$$\begin{cases} x = -0.18 \\ y = 0 \end{cases} \quad \begin{cases} x = -1.82 \\ y = 0 \end{cases}$$

$$\boldsymbol{x = 0}$$

$$y = 2 - 3(0+1)^2$$

$$y = -1$$

The coordinates of the point where the graph intersects the y-axis.

$$\begin{cases} x = 0 \\ y = -1 \end{cases}$$

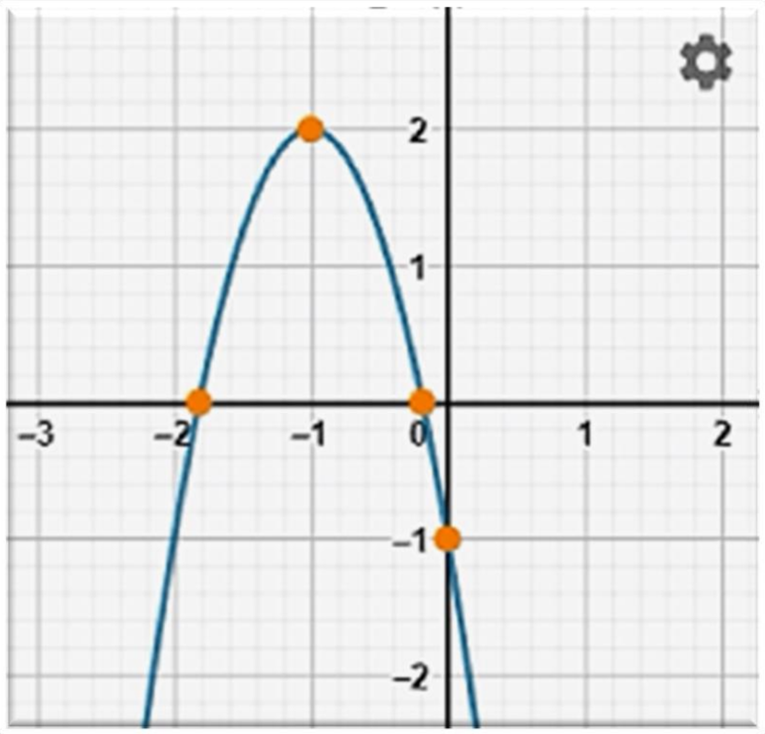
Example

Sketch the graph of $y = 2 - 3(x+1)^2$.

The coordinates of the turning point. $\begin{cases} x = -1 \\ y = 2 \end{cases}$

The coordinates of the points where the graph intersects the x-axis. $\begin{cases} x = -0.18 \\ y = 0 \end{cases}$ $\begin{cases} x = -1.82 \\ y = 0 \end{cases}$

The coordinates of the point where the graph intersects the y-axis. $\begin{cases} x = 0 \\ y = -1 \end{cases}$



Sketch cubic functions

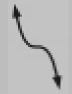
$$y = ax^3 + bx^2 + cx + d \qquad \text{or} \qquad f(x) = ax^3 + bx^2 + cx + d$$

This function is called **the cubic function.**

To sketch a cubic function, use the following steps.

1. Find the intersection of the graph with the coordinate axes.
   - To find the x-axis intercept of a function, substitute $y = 0$ and solve for x.
   - To find the y-axis intercept of a function, substitute $x = 0$ and solve for y.

2. Find the stationary points
   - Find the $f'(x)$.
   - Solve the equation $f'(x) = 0$
   - Substitute x in the function equation by the values of the found x-coordinate

3. Graph's shape
   - If $a > 0$ the graph's shape is  for two stationary point or  for one stationary point.
   - If $a < 0$ the graph's shape is  for two stationary point or  for one stationary point.

Example

Sketch the curve of $y = x^3 - 9x$.

Step 1

Find the intersection of the graph with the coordinate axes.

- To find the x-axis intercept of a function, substitute y = 0 and solve for x.
- To find the y-axis intercept of a function, substitute x = 0 and solve for y.

$\boldsymbol{y = 0}$

$y = x^3 - 9x$

$0 = x^3 - 9x$

$0 = x(x^2 - 9)$ → $x = 0$ or $x^2 - 9 = 0$

$x^2 = 9 \quad x = \pm 3$

The coordinates of the points where the graph intersects the x-axis.

$\begin{cases} x = -3 \\ y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\begin{cases} x = 3 \\ y = 0 \end{cases}$

$\boldsymbol{x = 0}$

$y = x^3 - 9x$

$y = (0)^3 - 9(0)$

$y = 0$

The coordinates of the point where the graph intersects the y-axis.

$\begin{cases} x = 0 \\ y = 0 \end{cases}$

Example

Sketch the curve of $y = x^3 - 9x$.

The coordinates of the points where the graph intersects the x-axis.

$\begin{cases} x = -3 \\ y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\begin{cases} x = 3 \\ y = 0 \end{cases}$

The coordinates of the point where the graph intersects the y-axis.

$\begin{cases} x = 0 \\ y = 0 \end{cases}$

Step 2 Find the stationary points

- Find the $f'(x)$.
- Solve the equation $f'(x) = 0$
- Substitute x in the function equation by the values of the found x-coordinate

$$y = x^3 - 9x$$

$$y' = 3x^2 - 9$$

$$3x^2 - 9 = 0$$

$$x^2 = 3 \quad \rightarrow \quad x = \pm\sqrt{3}$$

$$x = \sqrt{3} \quad \rightarrow \quad y = (\sqrt{3})^3 - 9\sqrt{3} = 10.39$$

$$x = -\sqrt{3} \quad \rightarrow \quad y = (-\sqrt{3})^3 - 9(-\sqrt{3}) = -10.39$$

Example

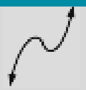
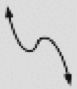
Sketch the curve of $y = x^3 - 9x$.

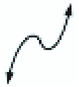
The coordinates of the points where the graph intersects the x-axis. $\begin{cases} x = -3 \\ y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\begin{cases} x = 3 \\ y = 0 \end{cases}$

The coordinates of the point where the graph intersects the y-axis. $\begin{cases} x = 0 \\ y = 0 \end{cases}$

stationary points $x = \sqrt{3}$ $y = 10.39$ $x = -\sqrt{3}$ $y = -10.39$

Step 3 Graph's shape

- If $a > 0$ the graph's shape is  for two stationary point or  for one stationary point.
- If $a < 0$ the graph's shape is  for two stationary point or  for one stationary point.

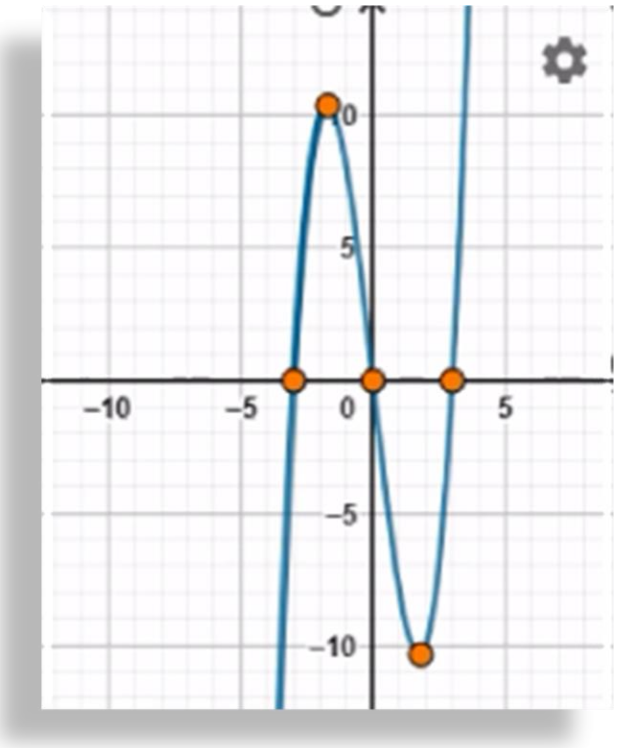
- $a = 1 > 0$ the graph's shape is  for two stationary point

Example Sketch the curve of $y = x^3 - 9x$.

The coordinates of the points where the graph intersects the x-axis. $\begin{cases} x = -3 \\ y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\begin{cases} x = 3 \\ y = 0 \end{cases}$

The coordinates of the point where the graph intersects the y-axis. $\begin{cases} x = 0 \\ y = 0 \end{cases}$

stationary points $x = \sqrt{3}$ $y = 10.39$ $x = -\sqrt{3}$ $y = -10.39$



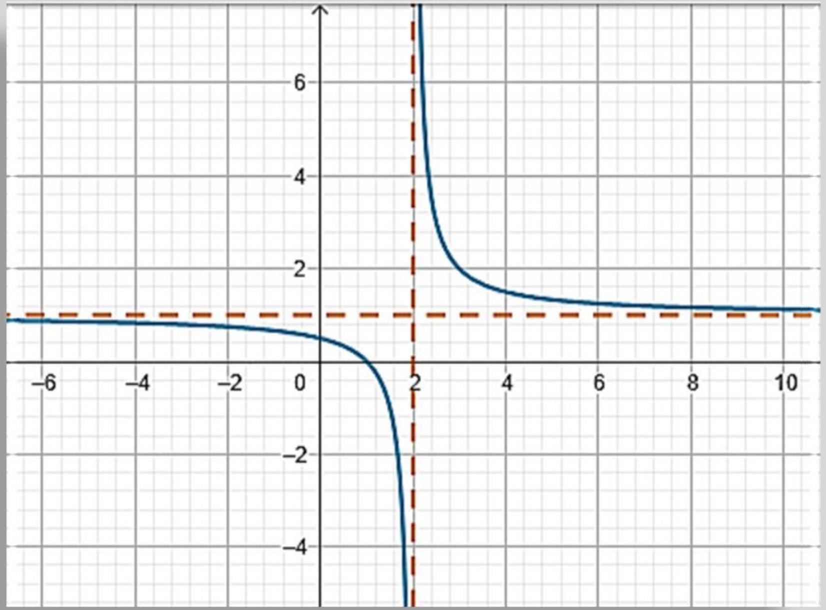
Asymptote

An **asymptote** is a straight line that a curve approaches but never touches or crosses as the curve extends to infinity. Asymptotes can be:

1) Horizontal asymptotes: The curve approaches a fixed horizontal line as $x \to \pm\infty$.

2) Vertical asymptotes: The curve approaches a vertical line as x approaches a specific value where the function is undefined.

$x = 2$ vertical asymptote



$y = 1$ horizontal asymptote

Sketch reciprocal function

$$y = \frac{a}{x} + c \quad \textbf{or} \quad f(x) = \frac{1}{x} + c$$

This function is called **the reciprocal function.**

The graph of a reciprocal function **is a hyperbola.**

The reciprocal functions have a vertical asymptote and a horizontal asymptote.

$x = 0$ The equation of vertical asymptote

$y = 0$ The equation of horizontal asymptote

To sketch the reciprocal function $y = \frac{1}{x}$

you always can use this table:

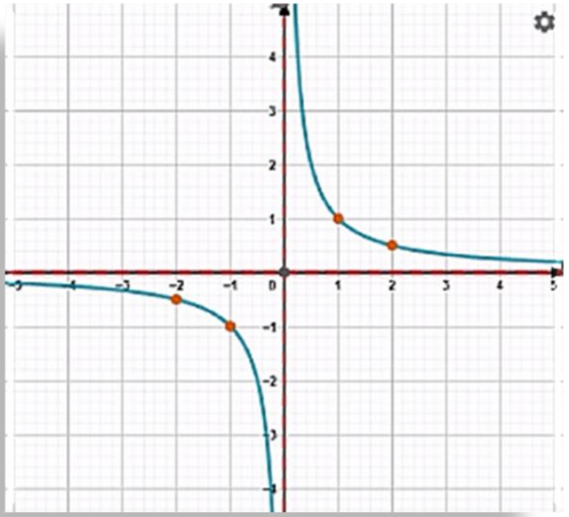
x	-2	-1	0	1	2
y	-0.5	-1	vertical asymptote	1	0.5

$$y = \frac{1}{x}$$

$$y = \frac{1}{-2} = -0.5$$

$$y = \frac{1}{x}$$

$$y = \frac{1}{-1} = -1$$



Sketch exponential function

$$f(x) = ab^x + c \qquad \text{or} \qquad y = ab^x + c$$

This function is called **the exponential function.**

The exponential functions have only a horizontal asymptote. $\boldsymbol{y = c}$

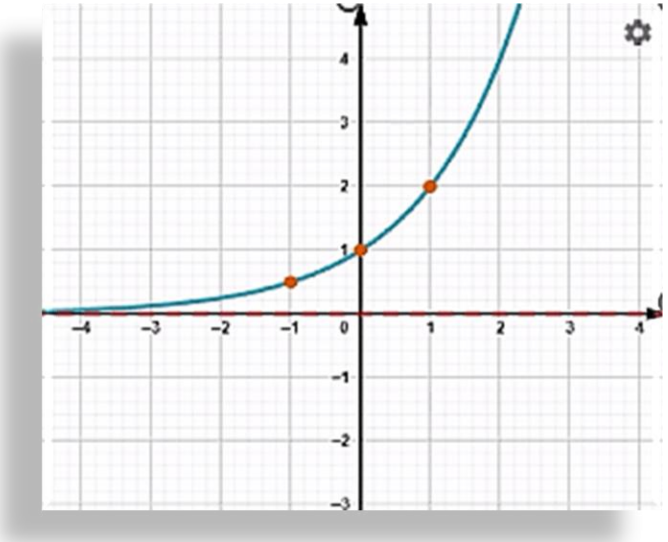
For example, if $a = 1, b = 2 \; and \; c = 0$, we construct tables of values and draw the graph $\boldsymbol{y} = 2^x$.

x	-1	0	1
y	0.5	1	2

$y = 2^x$
$y = 2^{-1}$
$y = \frac{1}{2} = 0.5$

$y = 2^x$
$y = 2^0$
$y = 1$

$y = 2^x$
$y = 2^1$
$y = 2$



The equation of horizontal asymptote $y = 0$