Adding and subtracting algebraic fractions

To add two or more fractions, we obtain the **lowest common denominator** and then add the resulting numerators.

To find the lowest common denominator, we look for the **lowest common multiple** of the denominators.

For example:

$$\frac{2x}{3}+\frac{5x}{2}=\frac{2x}{3}\times\frac{2}{2}+\frac{5x}{2}\times\frac{3}{3}=\frac{4x}{6}+\frac{15x}{6}=\frac{4x+15x}{6}=\frac{19x}{6}$$

To subtract two or more fractions, we obtain the **lowest common denominator** and then subtract the resulting numerators.

For example:

$$\frac{2x}{3}-\frac{5x}{2}=\frac{2x}{3}\times\frac{2}{2}-\frac{5x}{2}\times\frac{3}{3}=\frac{4x}{6}-\frac{15x}{6}=\frac{4x-15x}{6}=\frac{-11x}{6}$$

Example

Write as a single fractions in its simplest form.

$$\frac{1}{x}+\frac{5}{x+2}$$

$$\frac{1}{x}+\frac{5}{x+2} = \frac{1}{x}\times\frac{x+2}{x+2}+\frac{5}{x+2}\times\frac{x}{x}$$

$$= \frac{x+2}{x(x+2)}+\frac{5x}{x(x+2)}$$

$$= \frac{x+2+5x}{x(x+2)}$$

$$= \frac{6x+2}{x^2+2x}$$

Example

Write as a single fractions in its simplest form.

$$\frac{4}{x-1}+\frac{3}{x+1}$$

$$\frac{4}{x-1}+\frac{3}{x+1} = \frac{4}{x-1}\times\frac{x+1}{x+1}+\frac{3}{x+1}\times\frac{x-1}{x-1}$$

$$= \frac{4(x+1)}{(x-1)(x+1)}+\frac{3(x-1)}{(x+1)(x-1)}$$

$$= \frac{4x+4}{x^2-1}+\frac{3x-3}{x^2-1}$$

$$= \frac{4x+4+3x-3}{x^2-1}$$

$$= \frac{7x+1}{x^2-1}$$

Example

Write as a single fractions in its simplest form.

$$\frac{2}{2x-1}-\frac{5}{x+3}$$

$$\frac{2}{2x-1}-\frac{5}{x+3} = \frac{2}{2x-1}\times\frac{x+3}{x+3}-\frac{5}{x+3}\times\frac{2x-1}{2x-1}$$

$$= \frac{2(x+3)}{(2x-1)(x+3)}-\frac{5(2x-1)}{(x+3)(2x-1)}$$

$$= \frac{2x+6}{(2x-1)(x+3)}-\frac{10x-5}{(x+3)(2x-1)}$$

$$= \frac{2x+6-(10x-5)}{(2x-1)(x+3)}$$

$$= \frac{-8x+11}{(2x-1)(x+3)} = \frac{-8x+11}{2x^2+5x-3}$$

Multiplying and dividing algebraic fractions

To **multiply** two or more fractions, we multiply the numerators to form the new numerator, and we multiply the denominators to form the new denominator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

For example:

$$\frac{2x}{3y} \times \frac{5x}{7} = \frac{10x^2}{21y}$$

To **divide** by a fraction, we multiply by its reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

For example:

$$\frac{2x}{3y} \div \frac{5x}{7} = \frac{2x}{3y} \times \frac{7}{5x} = \frac{14\cancel{x}}{15\cancel{x}y} = \frac{14}{15y}$$

Example

Write as a single fractions in its simplest form.

$$\frac{1}{x} \times \frac{5x}{x+2}$$

$$\frac{1}{x} \times \frac{5x}{x+2} = \frac{5\cancel{x}}{\cancel{x}(x+2)}$$

$$= \frac{5}{x+2}$$

Example

Write as a single fractions in its simplest form.

$$\frac{2}{2x-1} \div \frac{5}{x+3}$$

$$\frac{2}{2x-1} \div \frac{5}{x+3} = \frac{2}{2x-1} \times \frac{x+3}{5}$$

$$= \frac{2 \times (x+3)}{(2x-1) \times 5}$$

$$= \frac{2x+6}{10x-5}$$

Simplifying algebraic fractions

If the numerator and denominator of an algebraic fraction are both written in factored form and common factors are found, we can simplify by cancelling the common factors.

Only factors can be canceled when cancelling in algebraic fractions, not terms.

For example:

$$\frac{\mathbf{2x + 6x^2}}{\mathbf{2x}} = \frac{2x(1 + 3x)}{2x} = 1 + 3x$$

It cannot be cancelled $2x$, because $2x$ in the numerator is a term.

Now we can cancel 2x because 2x is a factor.

For example:

$$\frac{\mathbf{(1 - 2x)^2}}{\mathbf{4 - 8x}} = \frac{(1 - 2x)(1 - 2x)}{4(1 - 2x)} = \frac{1 - 2x}{4}$$

Example

Simplify.

$$\frac{2x^2 + 5x + 3}{2x + 3}$$

$$\frac{2x^2 + 5x + 3}{2x + 3}$$

$2x^2 + 5x + 3$

$a \times c = 2 \times 3 = 6$

Factors of 6	6, 1	-6,-1	-2,-3	2,3
sum is equal to b.				2 + (3) = 5

$$2x^2 + 2x + 3x + 3 = 2x(x + 1) + 3(x + 1) = (x + 1)(2x + 3)$$

$$\frac{2x^2 + 5x + 3}{2x + 3} = \frac{(x + 1)(2x + 3)}{2x + 3}$$

$$= x + 1$$

Example

Simplify.

$$\frac{x^2-4x+3}{x^2-1}$$

$$\frac{x^2-4x+3}{x^2-1}$$

x^2-4x+3

$a \times c = 1 \times 3 = 6$

Factors of 3	3, 1	-3,-1
sum is equal to b.		$-3+(-1)=-4$

$$x^2-x-3x+3 = x(x-1)-3(x-1) = (x-1)(x-3)$$

$$\frac{x^2-4x+3}{x^2-1} = \frac{(x-1)(x-3)}{(x-1)(x+1)}$$

$$= \frac{(x-3)}{(x+1)}$$

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