Equation involving fractions

When solving equations involving fractions, multiply both sides of the equation by a suitable number or term to eliminate the fraction.

For example: Solve for x: $\frac{3+x}{x} = 4$

$$\frac{3+x}{x} = 4$$

$$\frac{3+x}{x} \times x = 4 \times x$$

$$3 + x = 4x$$

$$3 + x - x = 4x - x$$

$$\frac{3}{3} = \frac{3x}{3} \quad \rightarrow \quad 1 = x$$

Example

Find x:

$$\frac{1+x}{2}=\frac{1-x}{3}$$

$$\frac{1+x}{2}=\frac{1-x}{3}$$

$$\frac{1+x}{\cancel{2}}\times\cancel{2}=\frac{1-x}{3}\times 2$$

$$1+x=\frac{1-x}{3}\times 2$$

$$(1+x)\times 3=\frac{1-x}{\cancel{3}}\times 2\times\cancel{3}$$

$$3+3x=(1-x)\times 2$$

$$3+3x=2-2x$$

$$3+3x+2x=2-2x+2x$$

$$3+5x=2$$

$$3+5x-3=2-3$$

$$5x=-1$$

$$\frac{5x}{5}=\frac{-1}{5}$$

$$x=\frac{-1}{5}$$

Example

Solve the following equations:

a) $\frac{x}{2}+1=\frac{1}{4}$

b) $3\left(\frac{x}{2}+1\right)=4$

a) $\frac{x}{2}+1=\frac{1}{4}$

$\frac{x}{2}+1-1=\frac{1}{4}-1$

$\frac{x}{2}=\frac{1}{4}-\frac{4}{4}$

$\frac{x}{2}=\frac{-3}{4}$

$\frac{x}{2}\times 2=\frac{-3}{4}\times 2$

$x=\frac{-3}{2}$

b) $3\left(\frac{x}{2}+1\right)=4$

$\frac{3x}{2}+3=4$

$\frac{3x}{2}+3-3=4-3$

$\frac{3x}{2}=1$

$\frac{3x}{2}\times\frac{2}{3}=1\times\frac{2}{3}$

$x=\frac{2}{3}$

Example

Find x:

$$\frac{1+x}{2-x}=2$$

$$\frac{1+x}{2-x}=2$$

$$\frac{1+x}{\cancel{2-x}}\times\cancel{(2-x)}=2\times(2-x)$$

$$1+x=2\times(2-x)$$

$$1+x=4-2x$$

$$1+x+2x=4-2x+2x$$

$$1+3x=4$$

$$1+3x-1=4-1 \qquad 3x=3 \qquad \frac{3x}{3}=\frac{3}{3} \qquad x=1$$

Example

Find x:

$$\frac{x}{2x+1} = 4$$

$$\frac{x}{2x+1} = 4$$

$$\frac{x}{2x+1} \times (2x+1) = 4 \times (2x+1)$$

$$x = 4 \times (2x+1)$$

$$x = 8x + 4$$

$$x - x = 8x + 4 - x$$

$$0 = 7x + 4$$

$$0 - 4 = 7x + 4 - 4$$

$$\frac{-4}{7} = \frac{7x}{7} \quad \rightarrow \quad \frac{-4}{7} = x$$

Example

Find x:

$$\frac{1}{x+2} = \frac{3}{x-6}$$

$$\frac{1}{x+2} = \frac{3}{x-6}$$

$$\frac{1}{x+2} \times (x+2) = \frac{3}{x-6} \times (x+2)$$

$$1 = \frac{3}{x-6} \times (x+2)$$

$$1 \times (x-6) = \frac{3}{x-6} \times (x+2) \times (x-6)$$

$$x - 6 = 3 \times (x+2)$$

$$x - 6 = 3x + 6$$

$$x - 6 - x = 3x + 6 - x$$

$$-6 = 2x + 6$$

$$-6 - 6 = 2x + 6 - 6 \qquad \frac{-12}{2} = \frac{2x}{2} \qquad \rightarrow \qquad -6 = x$$

Example

Find x:

$$\frac{2}{x+2}+\frac{3}{2x-1}=0$$

$$\frac{2}{x+2}+\frac{3}{2x-1}=0$$

$$\frac{2}{x+2}=-\frac{3}{2x-1}$$

$$\frac{2}{x+2}\times(x+2)=-\frac{3}{2x-1}\times(x+2)$$

$$2=-\frac{3}{2x-1}\times(x+2)$$

$$2\times(2x-1)=-\frac{3}{2x-1}\times(x+2)\times(2x-1)$$

$$4x-2=-3\times(x+2)$$

$$4x-2+3x=-3x-6+3x$$

$$7x-2=-6$$

$$7x-2+2=-6+2 \qquad \frac{7x}{7}=\frac{-4}{7} \qquad \rightarrow \qquad x=\frac{-4}{7}$$

Example

Find x:

$$\frac{1}{2-x}+1=\frac{1}{4}$$

$$\frac{1}{2-x}+1=\frac{1}{4}$$

$$\frac{1}{2-x}+1-1=\frac{1}{4}-1$$

$$\frac{1}{2-x}=\frac{1}{4}-\frac{4}{4}$$

$$\frac{1}{2-x}=\frac{-3}{4}$$

$$\frac{1}{\cancel{2-x}}\times\cancel{(2-x)}=\frac{-3}{4}\times(2-x)$$

$$1=\frac{-3}{4}\times(2-x)$$

$$1\times 4=\frac{-3}{4}\times(2-x)\times 4$$

$$4=-3\times(2-x)$$

$$4=-6+3x$$

$$4+6=-6+3x+6$$

$$10=3x$$

$$\frac{10}{3}=x$$

$$3\frac{1}{3}=x$$

$$\frac{2}{x-1}+\frac{x}{x+1}=1$$

$$(x-1)(\frac{2}{x-1}+\frac{x}{x+1})=1(x-1)$$

$$2+\frac{x(x-1)}{x+1}=(x-1)$$

$$(x+1)(2+\frac{x(x-1)}{x+1})=(x-1)(x+1)$$

$$2(x+1)+x(x-1)=(x-1)(x+1)$$

$$2x+2+x^2-x=x^2-1$$

$$x+2=-1$$

$$x=-3$$

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