Percentage of a quantity

To find R% of n you just need to multiply $\frac{R}{100}$ with n.

For example:

Find the Value of 15% of 1250.

$$\frac{15}{100} \times 1250 = 187.5$$

Example:

In a basket of apples, 25% are spoiled and 60 are in good condition. Find the total number of apples in the basket.

25% are spoiled $\Rightarrow 1 - \frac{25}{100} = \frac{75}{100}$ are good

$$\frac{75}{100} \times x = 60 \longrightarrow \frac{75}{100} \times x \times \frac{100}{75} = 60 \times \frac{100}{75}$$

$$x = 80$$

Express one quantity as a percentage of another

To express m as a percentage of n then the formula is:

$$\text{m as a percentage of n} = \frac{m}{n} \times 100$$

Note:

- **While Expressing One Quantity as a Percentage of Another the two quantities must be of the same kind and should have the same units.**

For example:

Express 2 Hours 10 Minutes as a Percentage of 4 Hours 20 Minutes.

2 Hour 10 Minutes = 130 Minutes

4 Hour 20 Minutes = 260 Minutes

$$\frac{130}{260} \times 100 = 50\%$$

Example

Write 26 g as a percentage of 208 g.

$$\frac{26}{208} \times 100 = 12.5\%$$

Example

A shop sold 500 shirts last month. 41% of these shirts were sold online and the rest were sold in stores.

Calculate the number of shirts sold in the store.

$$1 - \frac{41}{100} = \frac{59}{100}$$

Were sold in stores

$$500 \times \frac{59}{100} = 295$$

The number of shirts sold in the store.

Example

John is buying a car worth $12,000. The dealership requires a deposit of 15% of the car's price.

a) How much is the deposit?
b) How much will John still need to pay after the deposit?

$a)$

$$Deposit = \frac{15}{100} \times 12000 = 1800$$

$$Deposit = 1800$$

$b)$

$$Remaining\ Amount = Car\ Price - Deposit\ Remaining\ Amount$$

$$Remaining\ Amount = 12000 - 1800 = 10200$$

Final amount after the R% increase or decrease

Final amount after the R% increase

$$Final\ amount = initial\ amount \times \left(1 + \frac{R}{100}\right)$$

For example:

Adeline invests $550 in an account paying simple interest at rate 3.3% per year.
Calculate the value of her investment at the end of year.

value of her investment at the end of year $= 550 \times \left(1 + \frac{3.3}{100}\right)$

$= \$568.15$

Final amount after the R% increase or decrease

Final amount after the R% increase

Final amount after the R% decrease

$$Final\ amount\ =\ initial\ amount\ \times (1 - \frac{R}{100})$$

For example:

Alfred buy a car for $50000. He sells it at loss of 23%.
Calculate the selling price of the car.

$$\text{value of the selling price} = 50000 \times (1 - \frac{23}{100})$$

$$= \$38,500$$

Example

Olivia buy a car for $27000. She sells it at loss of 17%.

Calculate the selling price of the car.

$$\boldsymbol{Final\ amount = initial\ amount \times (1 - \frac{R}{100})}$$

$$Final\ amount = 27000 \times (1 - \frac{17}{100})$$

$$Final\ amount = \$22410$$

Example

The cost of a bus travel is increase by 5% to a new cost of $95.

Calculate the original cost of the bus travel.

$$Final\ amount = initial\ amount \times (1 + \frac{R}{100})$$

$$95 = x \times (1 + \frac{5}{100})$$

$$95 = 1.05x$$

$$\frac{95}{1.05} = x$$

$$\$90.48 = x$$

Example

The number of students in a school in the last semester was 270. The number of students in this school this semester is 378.

Calculate the percentage increase in the number of students.

$$\textit{Final amount} = \textit{initial amount} \times \left(1 + \frac{R}{100}\right)$$

$$378 = 270\left(1 + \frac{R}{100}\right)$$

$$378 = 270 + \frac{270R}{100}$$

$$378 - 270 = 2.7R$$

$$\frac{108}{2.7} = \frac{2.7R}{2.7}$$

$$40 = R$$

Percentage increase in the number of students = 40%

Example

In a shop in one-month sales increase by 12%.
The following month sales increased by 15%.

Calculate the overall percentage increase in sales.

Regular sales= x

$$\textbf{Final amount} = \textbf{initial amount} \times (1 + \frac{R}{100})$$

First month sales $= x(1 + \frac{12}{100})$

$= 1.12x$

$$\textbf{Final amount} = \textbf{initial amount} \times (1 + \frac{R}{100})$$

Second month sales $= 1.12x(1 + \frac{15}{100})$

$= 1.288x$

$$\textbf{Final amount} = \textbf{initial amount} \times (1 + \frac{R}{100})$$

$$1.288x = x\left(1 + \frac{R}{100}\right)$$

$$\frac{1.288x}{x} = \frac{x\left(1 + \frac{R}{100}\right)}{x}$$

$$1.288 - 1 = 1 + \frac{R}{100} - 1$$

$$0.288 \times 100 = \frac{R}{100} \times 100$$

$$28.8 = R$$

Overall percentage increase in sales = 28.8%

Example

Alfred invests some money.
By the end of the first year, the value of the investment has decreased by 25%.
By the end of the second year, the value of the investment has increased by 40% of its value at the end of the first year.

Calculate the overall percentage change in the value of the investment.

Initial investment $= x$

$$\boldsymbol{Final\ amount = initial\ amount \times \left(1 - \frac{R}{100}\right)}$$

By the end of the **first** year, the value of the investment $= x\left(1 - \frac{25}{100}\right)$

$$= 0.75x$$

$$\boldsymbol{Final\ amount = initial\ amount \times \left(1 + \frac{R}{100}\right)}$$

By the end of the **second** year, the value of the investment $= 0.75x\left(1 + \frac{40}{100}\right)$

$$= 1.05x$$

$$\boldsymbol{Final\ amount = initial\ amount \times \left(1 + \frac{R}{100}\right)}$$

$$1.05x = x\left(1 + \frac{R}{100}\right)$$

$$\frac{1.05x}{x} = \frac{x\left(1 + \frac{R}{100}\right)}{x}$$

$$1.05 - 1 = 1 + \frac{R}{100} - 1$$

$$0.05 \times 100 = \frac{R}{100} \times 100$$

$$5 = R$$

Overall percentage increase in sales $= 5\%$

Percentages over 100%

Percentages over 100% describe situations where a quantity exceeds the original or base value.
A percentage represents a fraction out of 100 so, percentages greater than 100% imply more than the whole.

For example:

Growth or Increase:
If a population grows by 150%, it means the final population is 1.5 times the initial population.

Profit and Loss:
A profit of 200% means the profit is twice the original cost.

Excess Quantity:
If you drink 110% of your daily water requirement, you've consumed 10% more than required.

Example

A shopkeeper increases the price of a product by 125%.

If the original price was $40, find the new price.

$$\textit{Final amount} = \textit{initial amount} \times \left(1 + \frac{R}{100}\right)$$

$$\textit{Final price} = 40 \times \left(1 + \frac{125}{100}\right)$$

$$\textit{Final price} = 90\ \$$$

Interest

Interest

When money is lent, the individual making the loan and the person receiving it are referred to as the lender and borrower, respectively.

The term "**principal**" refers to the amount borrowed from the lender.

The borrower is typically assessed with a fee known as interest by the lender. This fee represents the expense of using someone else's money. The borrower is required to pay back the borrowed principal amount plus **interest**.

The principal, the time the money is borrowed, and the interest rate all influence how much interest is charged on a loan.

Interest

Interest

Simple interest

In this method for the duration of the loan, interest is calculated using the simple interest technique on the initial amount borrowed.

$$Final\ amount\ =\ initial\ amount\ \times (1+\frac{R}{100}\times n)$$

The total of money returned after the borrowing period is known as the final amount.

r is the rate of interest per annum as a decimal
n is the number of periods or duration of the loan in years.

Note:

- $\boldsymbol{Total\ Interest\ Earned\ = Final\ amount - initial\ amount\ (Principal)}$

Interest

Interest

Simple interest

Compound interest

When money is kept in a bank for a while, interest is automatically applied to the account, increasing the principal.
Next, the higher principal will be used to determine a subsequent lot of interest.
Since you are receiving interest on interest, this has the effect of compounding the interest.

$$Final\ amount\ =\ initial\ amount \times \left(1 + \frac{R}{100}\right)^n$$

The total of money returned after the borrowing period is known as the final amount.

r is the rate of interest per annum as a decimal
n is the number of periods or duration of the loan in years.

Note:

- $\boldsymbol{Total\ Interest\ Earned\ = Final\ amount - initial\ amount\ (Principal)}$

Example

Alfred invests $550 in an account paying simple interest at rate 3.3% per year.

Calculate the value of his investment at the end of 5 years.

$$\boldsymbol{Final\ amount} = \boldsymbol{initial\ amount} \times \left(\mathbf{1} + \frac{\boldsymbol{R}}{\mathbf{100}} \times \boldsymbol{n}\right)$$

value of his investment at the end of 5 years $= 550 \times \left(1 + \frac{3.3}{100} \times 5\right)$

$= \$\ 640.75$

Example

Alfred invests $550 for 5 years in an account paying compound interest at rate 3.3% per year.

Calculate the value of his investment at the end of 5 years.

$$\boldsymbol{Final\ amount = initial\ amount \times \left(1 + \frac{R}{100}\right)^{n}}$$

$$\text{value of his investment at the end of 5 years} = 550 \times \left(1 + \frac{3.3}{100}\right)^{5}$$

$$= \$\ 646.94$$

Example

Alfred invests \$1000 for 6 years in an account paying simple interest at rate 2.1% per year.

Calculate the total interest earned during the 6 years.

$$\textbf{Final amount} = \textbf{initial amount} \times \left(1 + \frac{R}{100} \times n\right)$$

value of his investment at the end of 6 years $= 1000 \times \left(1 + \frac{2.1}{100} \times 6\right)$

$= \$\ 1126$

- $\textbf{Total Interest Earned} = \textbf{Final amount} - \textbf{initial amount (Principal)}$

$Total\ Interest\ Earned = 1126 - 1000$

$= \$\ 126$

Example

Alfred invests $1000 for 6 years in an account paying compound interest at rate 2.1% per year.

Calculate the total interest earned during the 6 years.

$$\textbf{Final amount} = \textbf{initial amount} \times \left(1 + \frac{R}{100}\right)^{n}$$

value of his investment at the end of 6 years $= 1000 \times \left(1 + \frac{2.1}{100}\right)^{6}$

$= \$\ 1132.80$

- $\textbf{Total Interest Earned} = \textbf{Final amount} - \textbf{initial amount}\ (\textbf{Principal})$

$Total\ Interest\ Earned = 1132.80 - 1000$

$= \$\ 132.80$

mathematicschapters.com